# LEARNING MATERIAL OF STRUCTURAL MECHANICS

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**&**

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# Review of basic concept

## Shear force & bending moment of beam.

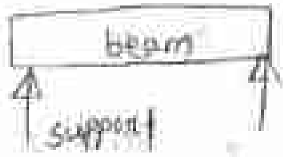
* **Beam** :- It is a horizontal member which transmits the lateral load.


* what is the objective of beam ?
→ The objective of beam is to transmit the lateral load to the beam and finally to the use of beam.
→ The beam are used in framed structer Bipe, water tank, cable, rack, etc.

* What is structure:-
is a body → several element such as beams columns slabs etc.
→ which can sit up resistance against deformat by the application of external force.

* Types of beam :-
① simple supported beam
② Cantilever beam
③ overhanging beam
④ Fixed beam
⑤ Contineous beam

Simple supported beam :- A beam supported on resting freely on support is known as simple supported beam.



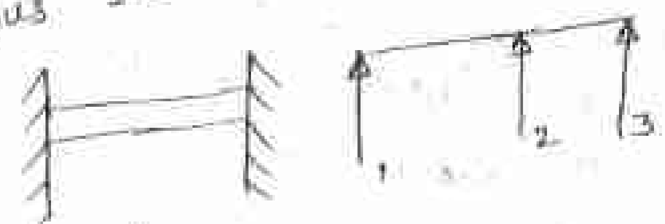

② Cantilever beam:- A beam whose one end is fixed and other end is free is known as cantilever beam.


③ over hanging beam:- If end portion of the beam extended beyond the support is known as over hanging beam.


④ fixed beam:- The beam whose both ends are fixed is known as fixed beam.

→ It is also called as built in beam or end beam.

⑤ Continious beam:- If the beam having more than two support is known as continious beam.




Types of load

① point load
② uniformly distributed load (u.d.L)

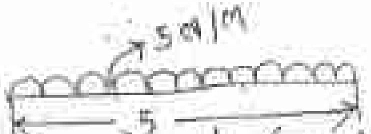
(3) uniformly varying load (u.v.l)

① Point load :- The load which acts at a point is known as point load.

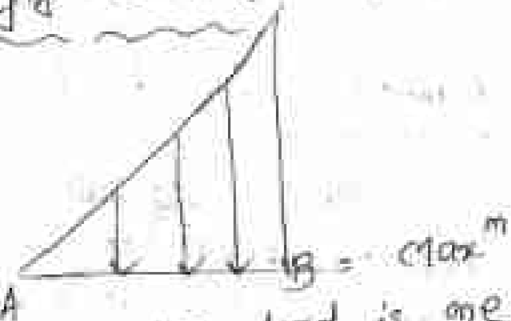
→ The unit of point Bn or kn (↓)

② uniformly distributed load (u.d.l)

→ A uniformly distributed is one which spead over the entire length of beam in such way the rate of loading is uniform.




③ uniformly varying load (u.d.l)




→ A uniformly varying load is one which spead over the entire length of the beam in such way that the rate of loading is non - uniform.

→ It is expressed as n/m

Types of support :-

(1) simple support

② hinge support

Rosetom deul uped

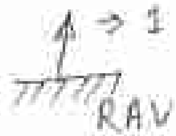
No of unknown Reaction

→ 4 → 1 (RAH Horizontal)

→ 4 → RAV (votical) → 2

④ Roller support

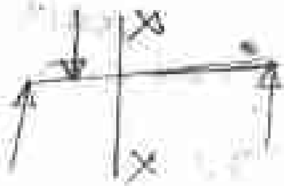
symbol




⑤ Fixed support

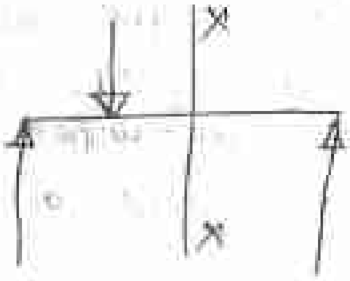

* (S.F) Shear force :-

The algebric sum all forces either left or right part of the beam section is known as shear force.




* Bending moment B.M

The algebric sum of all moment either left or right part of the beam section is known as Bending moment.

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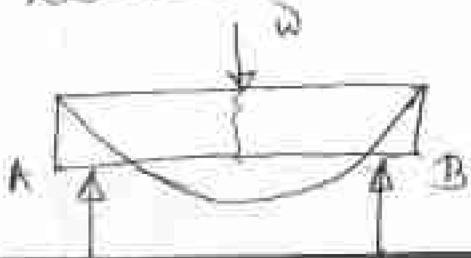
Bending moment & shear force

Bending (B.M)

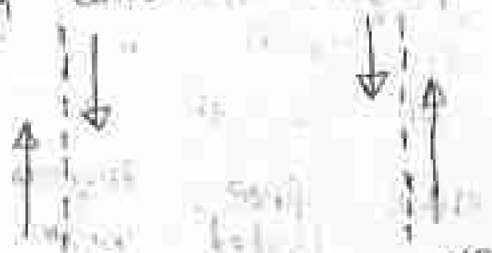


Defⁿ of shear force

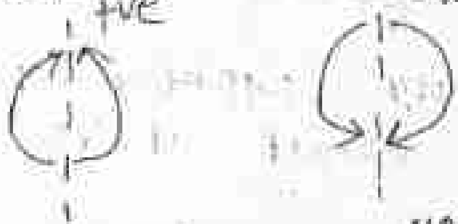
→ The algebraic sum all forces either left or right part of the beam section.


B·M The algebraic sum of all moments either left or right part of the beam section is known as Bending.


VVIMT sign convention of S·f




sign convention of B·M




moment = Force × ⊥ distance

$M = F \times \perp \text{distance}$ $\begin{bmatrix} N-M \\ KN-M \end{bmatrix}$ unit

$M = F \times 0 = 0$

⑥ Shear force diagram :- (S.F.D) :

The shear force diagram is one which shows the variation of S.F along length of the beam.

Bending moment diagram (B.M.D)

Bending moment diagram is one which shows the variations of Bending moment along the length of the beam.

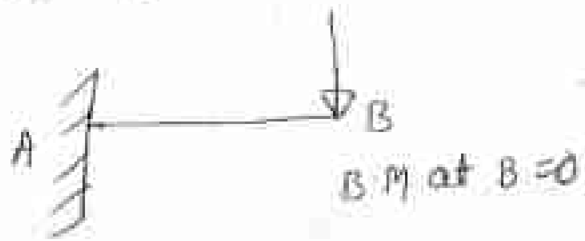
IMP points to be Remembered while drawing the B.M.D & S.F.D

(1) The length of the bending moment diagram & S.F.D must be equal to the length of the beam.

(2) The shear force diagram is drawn below the loaded beam diagram & B.M.D is drawn below the shear force diagram.

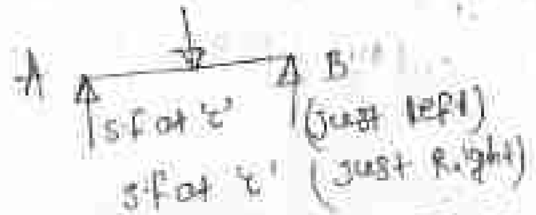
(3) In simply supported beam the bending moment at its ends is zero.


(4) In cantilever beam, the bending moment at it's free end is zero.

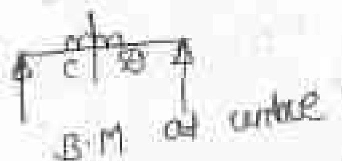


(5) Calculate S.F and B.M at every critical section.

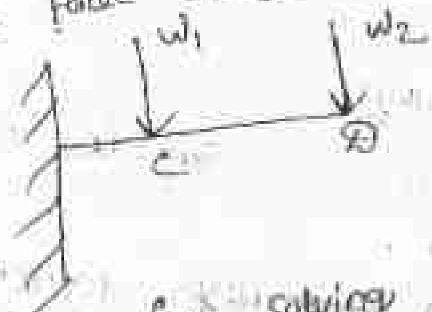
(6) If a point Load is acting then S.F is calculated Just left and Just Right.



(7) If u.d.l is acting then At it's both ends B.M & S.F is to be calculated.



(8) If there is no load acting betn two load the shear force diagram is constant.

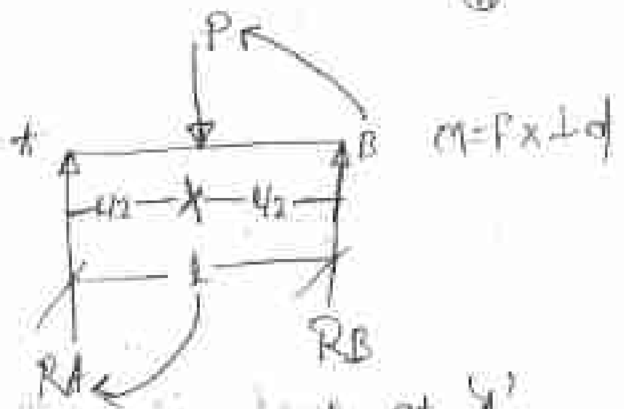


Procedure for solving S.F and B.M problem.

(1) Incase of simply supported beam calculate Reaction at it's supports.

(2) In case of cantilever calculate S.F at every critical section.

Example :-




Taking moment at 'A'

$\Sigma A \cdot M = T \cdot C \cdot M \Rightarrow R_B \times l = Pl/2$

(Total Anticlockwise Moment)

$R_B = \frac{P}{2}$

$T \cdot U \cdot L = T \cdot L \cdot D \; T \cdot D \cdot L$

$R_A + R_B = P$

$R_A = P - R_B$

$= P - \frac{P}{2}$

$= P/2$

18 Aug 2020 — Bending moment & shear force

① Statically.

A structure can be analysed by two method.

(i) Statically determinate structure

(ii) Statically indeterminate structure

① Statically determinate structure :-

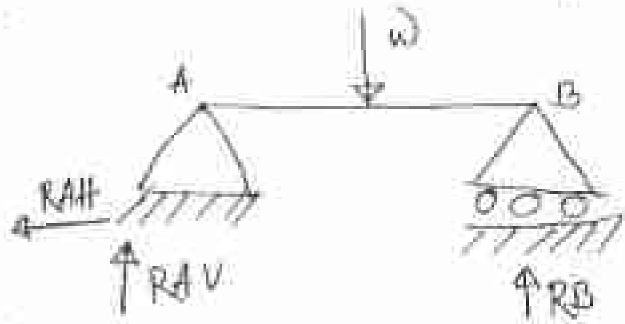
If a structure can be analysed by eq^n of equilibrium.

(i) $\Sigma F_X = 0$

(ii) $\Sigma F_Y = 0$

(iii) $\Sigma M_Z = 0$

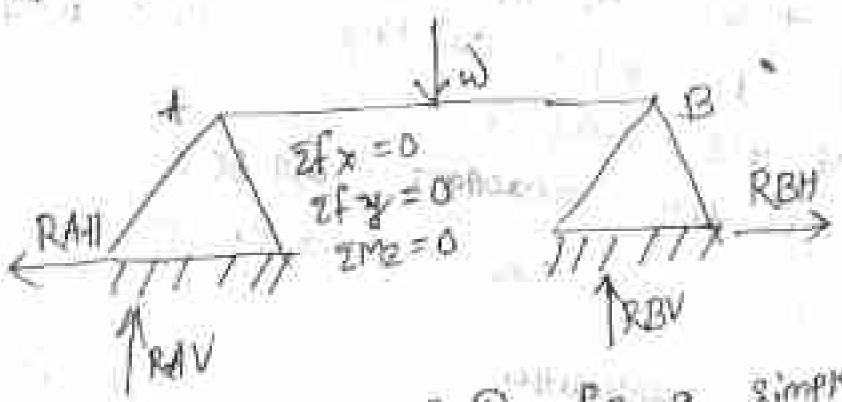
eq^n of equilibrium



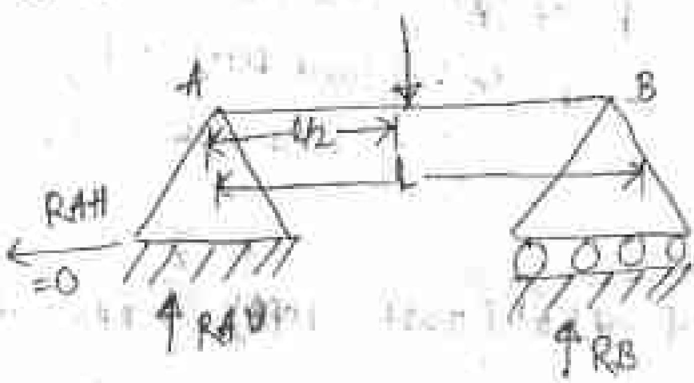

Here the no of reactions are = 3

(ii) Statically indeterminate structure :-

If a structure can't be solved by eqn of equilibrium.




Draw B.M.D. & S.F.D for a simple supported carrying a point load at it's centre.




$M = f \times \perp d$
$= R_{AH} \times 0$
$\Rightarrow M = 0$

Taking moment at 'A'

$$R_B \times l = w \times \frac{l}{2}$$

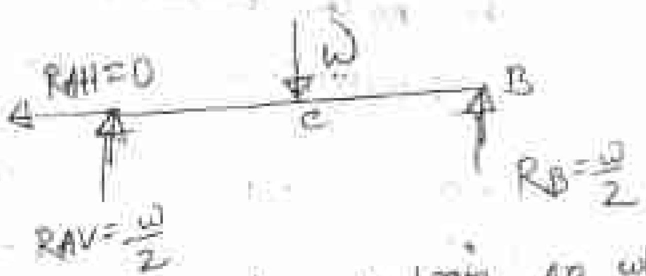
$$\Rightarrow R_B = \frac{w}{2}$$

(10) Total upward Load = Total downward load

$$R_{AV} + R_B = W$$

$$\Rightarrow R_{AV} = W - \frac{W}{2}$$

$$= \frac{W}{2}$$




Let us consider a beam AB whose Length is 'L' and carryings a point load (W) at its centre.

So the reactions will be

$$R_{AV} = \frac{W}{2}, \; R_B = \frac{W}{2}$$

S.F calculation :-

S.F at 'A' (Just left) = 0

S.F at 'A' (Just Right) = $+\frac{W}{2}$

S.F at 'C' (Just left) = $+\frac{W}{2}$

(Just right) = $R_{AV} - W$

$$= \frac{W}{2} - W = -\frac{W}{2}$$

S.F at 'B' (Just left) = $R_{AV} - W$

$$= \frac{W}{2} - W = -\frac{W}{2}$$

S.F at 'B' (Just Right) = $R_{AV} + R_B - W$

$$= \frac{W}{2} + \frac{W}{2} - W$$

$$= 0$$

Bending moment calculation :-

Bending moment 'A' = 0

" " 'C' = $+R_{AV} \times l/2$

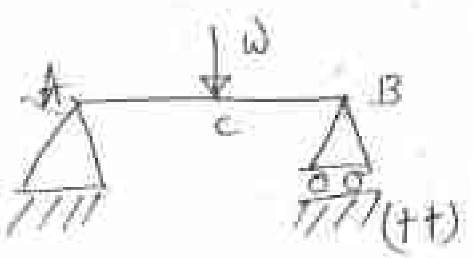
$= +\left(\frac{w}{2} \times \frac{l}{2}\right)$

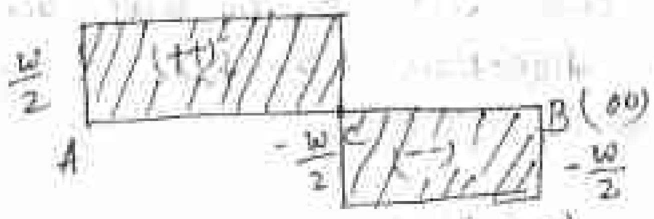
$(B \cdot M)_{(Maximum)} = +\frac{wl}{4}$

Bending at 'B' = $R_{AV} \times l - w \times \frac{l}{2}$

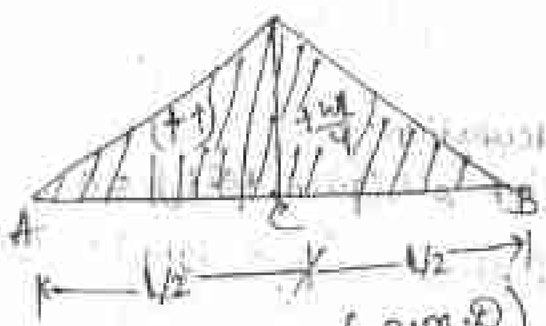
$= \frac{w}{2} \times l - \frac{wl}{2}$

$= '0'$




(S.F.D) (--)



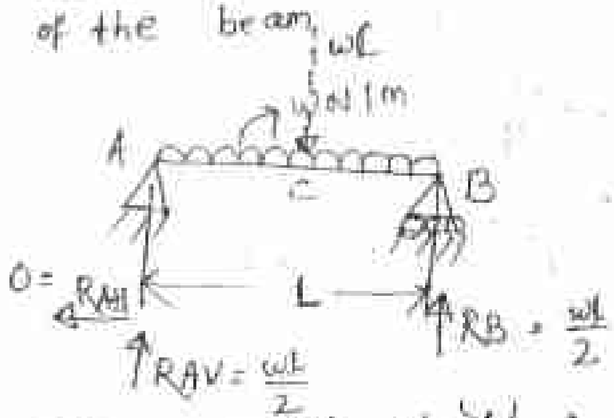

(B.M.D)

(W.V.IMP)

(Maximum B.M occured where the shear force is zero)

19 Aug 2020

→ A simply supported beam carrying uniformly distributed load throughout the length of the beam.




Taking moment at 'A':

T.A.M = T.C.M

$\Rightarrow R_B \times L = w \times L \times \frac{L}{2}$

$\Rightarrow \boxed{R_B = +\frac{wL}{2}}$

We know that there is no load acting in horizontal direction so $R_{AH} = 0$, we have to calculate $R_{AV}$:

T.U.L = T.D.L

$\Rightarrow R_B + R_{AV} = wL$

$\Rightarrow R_{AV} = wL - \frac{wL}{2} = \frac{wL}{2}$

S.F calculation:-

S.F At 'A' (Just left) = 0

S.F At 'A' (Just right) = $+\frac{wL}{2}$

S.F At 'C' = $\frac{wL}{2} - \frac{wL}{2} = 0$

S.F at 'B' (Just left)

$= \frac{wL}{2} - wL = -\frac{wL}{2}$

S.F. at 'B' (J.R) $= \frac{wL}{2} - \frac{wL}{2} - wL = 0$

Bending moment calculation -

$B.M$ at 'A' $= 0$

$B.M$ at 'B' $= 0$

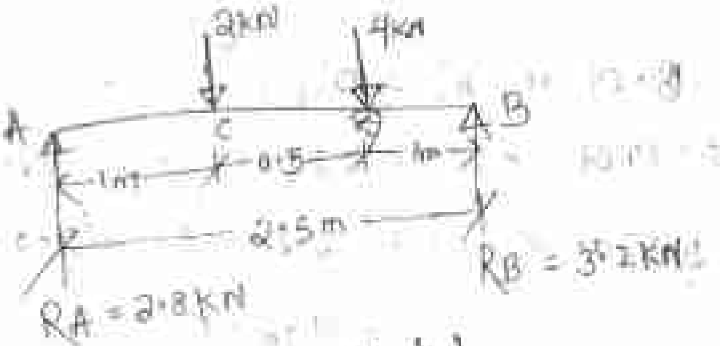
$B.M$ at 'C' $= R_{AV} \times \frac{l}{2} - \frac{wl}{2} \times \frac{l}{4}$

$\Rightarrow \frac{wl}{2} \times \frac{l}{2} - \frac{wl^2}{8}$

$\Rightarrow \frac{wl}{2}$

25 Aug 2020

Prob-1 A simply supported beam of span 2.5m as shown in the figure. Draw Bending moment & shear force diagram of the given beam.




So Taking moment A)

$\Rightarrow T.A.C.M = T.C.C.M$

$\Rightarrow R_B \times 2.5 = 2 \times 1 + 4 \times 1.5$

$\Rightarrow R_B \times 2.5 = 2 + 4 \times 1.5$

$\Rightarrow \boxed{R_B = 3.2KN}$

II Total up load = Total downward load

$R_A + R_B = 2 + 4$

$\Rightarrow R_A = 6 - 3.2 = 2.8KN$

S.F Calculation :-

S.F at 'A' (J.L) = 0

S.F at 'A' (J.R) = +2.8 kN.

S.F at 'C' (J.L) = +2.8 kN.

S.F at 'C' (J.R) = +2.8 kN − 2

= 0.8 kN.

S.F at 'D' (J.L) = +2.8 − 2 = 0.8 kN

S.F at 'D' (J.R) = +2.8 − 2 − 4

= −3.2 kN

S.F at 'B' (J.L)

= +2.8 − 2 − 4 = −3.2 kN

S.F at 'B' (J.R) = (+2.8 + 3.2) − (2+4)

= 6 − 6 = 0

B.M at 'A' = 0

B.M at 'C' = + (2.8×1) = 2.8 kN−m

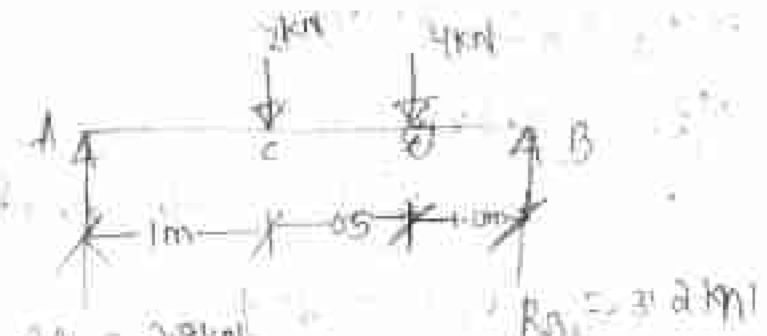
B.M at 'D' = + (2.8×1.5) − (2×0.5)

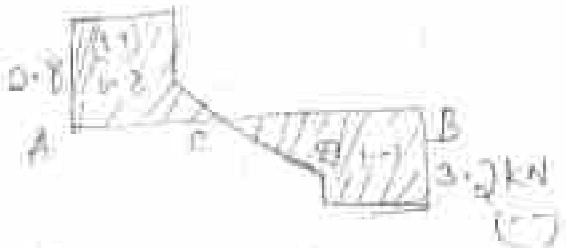
= 4.2 − 1

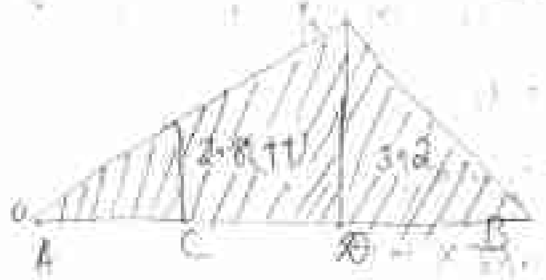
= 3.2 kN−m.

B.M at 'B' = (2.8×2.5) − (4×1 + 2×1.5)

= 0



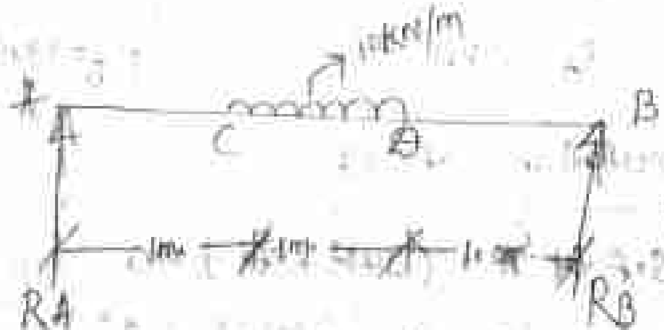




B.M.D

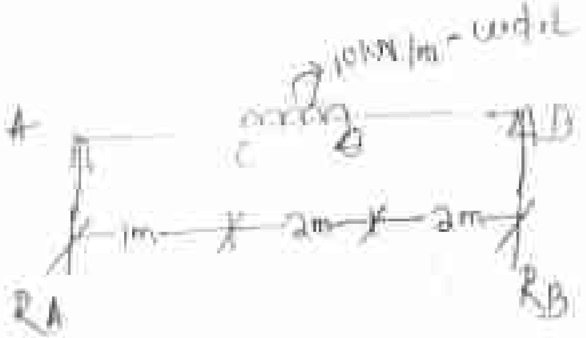
Prob-2

A simply supported beam is loaded given below. Find the point at which the bending moment will be max^m and also draw B.M.D & S.F.D the beam.



26 Aug 2020

Prob-2 Draw shear force and bending moment diagram for the beam indicating the value of max^m bending moment. The loaded diagram of the beam is given below.




Taking moment at 'A' to calculate reaction 'RB' $\Sigma M_A = 0$.

$\Rightarrow$ T.A.M = T.C.M

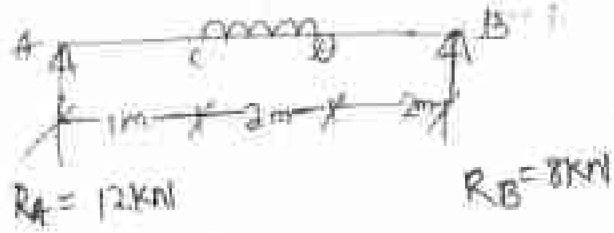
$\Rightarrow R_B \times 5 = 10 \times 2 \times (1+1)$

$\Rightarrow R_B = 40/5 = 8$ KN.

T.U.L = T.D.L

$\Rightarrow R_A + R_B = 10 \times 2$

$\Rightarrow R_A = 20 - 8 = 12$ KN




Calculation of SF :-

(i) S.F at 'A' (Just Left) = 0

(ii) S.F at 'A' (Just Right) = +12 KN.

(iii) S.F at 'C' = +12 KN

(iv) S.F at 'D' = +12 KN − (10×2)

= −8 KN

(v) S.F at 'B' (Just Left) = +12 − (10 × 2)

= − 8 KN .

S.f at 'B' (Just Right) $= (12+8) - (10\times2)$

$= 20 - 20 = 0$

<u>Bending moment cal$^n$:-</u>

Bending moment 'A' $= 0$

B.M at 'C' $= (12\times1) = 12$ kN-m

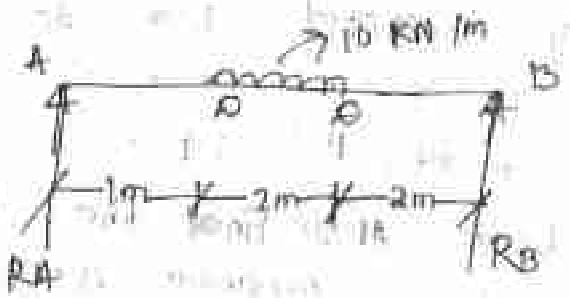
B.M at 'D' $= (12\times3) - (10\times2\times\frac{2}{2})$

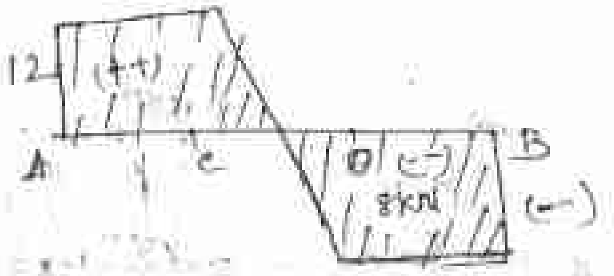
$= 16$ kN·m.

Bending moment at 'B' $= (12\times5) - (10\times2\times(\frac{2}{2}+2))$

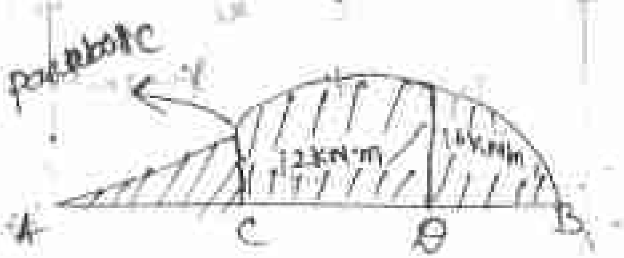
$= 60 - (20\times3)$

$= 60 - 60 = 0$



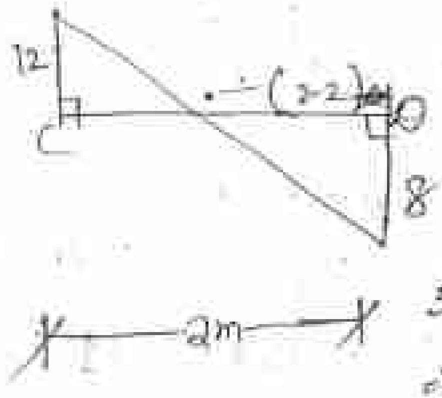




(S.f.D)




(B.M.D)

Let 'x' be drawn point from 'C'




$\frac{x}{12} = \frac{2-x}{8}$

$\Rightarrow 8x = 24-10$

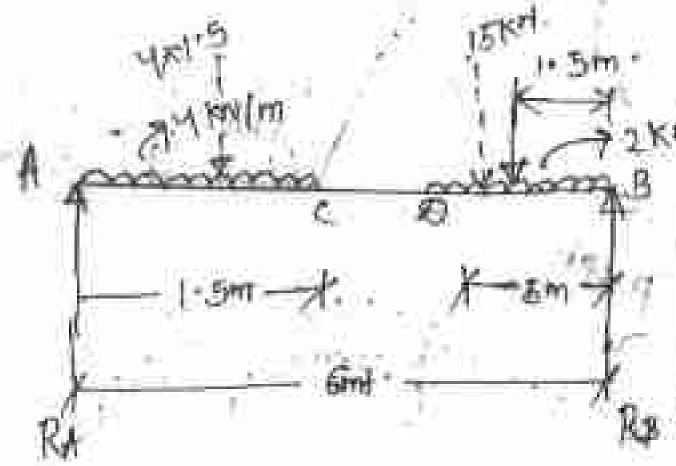
$\Rightarrow 20x = 24$

$\Rightarrow x = \frac{24}{20}$

$\Rightarrow 1.2\,m$

29 Aug 2020

Q) A simply supported beam AB = 6 m long is loaded as shown in the figure constant the S.F.D and B.M.D for the beam. Also find the position and volume of maximum B.M.




CD = 1.5m.
DB = 1.5m.
DB = 3m.
AB = 6m.

Step-1 Taking moment at A, $\Sigma M_A = 0$

Total anticlockwise moment

T.A.M = T.C.M (6-4.5 = 1.5m.)

$\Rightarrow R_B \times 6 = 4 \times 1.5 \times \frac{1.5}{2} + 5 \times 4.5 + 2 \times 3 \times (1.5 + 1.5 + 1.5)$

$\Rightarrow R_B = \frac{54}{6} = 9\ KN$

<u>step-2</u> Total upword load = total Downward load

T.U.L = T.D.L

$\Rightarrow R_A + R_B = 4 \times 1.5 + 5 + 2 \times 3$

$\Rightarrow R_A = 17 - R_B$

$= 17 - 9 = 8\ KN$

$\Rightarrow R_B = 9\ KN$

<u>step-3</u> shear force S.F calculation

S.F at 'A' (Just Left) = 0

S.F at 'A' (Just Right) = +8 KN

S.F at 'C' $= 8 - (4 \times 1.5)$

$= 8 - 6$

$= 2\ KN$

Shear force at 'E' (Just Left) $= 8 - (4 \times 1.5) - (2 \times 1.5)$

$= -1\ KN$

S.F at 'E' (Just Right) $= 8 - (4 \times 1.5) - (2 \times 1.5) - 5\ KN$

$= -6\ KN$

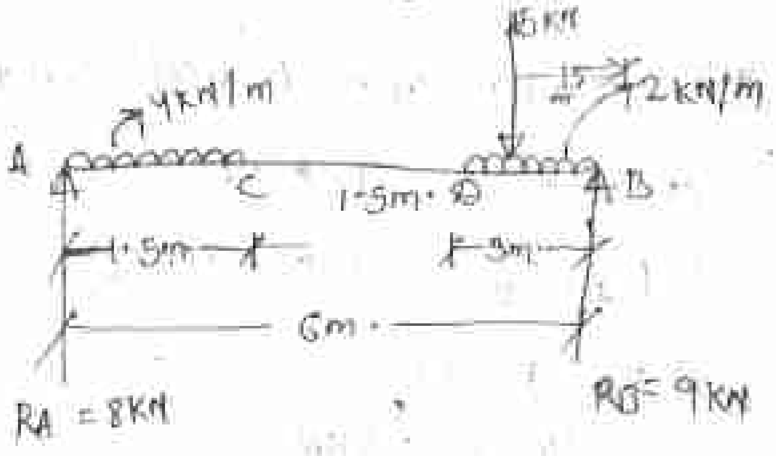
S.F at 'B' (J.L) $= 8 - (4 \times 1.5) - (2 \times 3) - 5\ KN$

$= -9\ KN$

S.F at 'B' (J.R) $= (8+9) - (4\times1.5) - (2\times3) - 5$

$= 17 - 6 - 6 - 5 = 17 - 17$

$= 0$




Step-4 Bending moment calculation

B.M at 'A' $= 0$

B.M at 'C' $= R_A \times 1.5 - \left(4\times1.5\times\frac{1.5}{2}\right)$

$= (8\times1.5) - (4\times1.5\times0.75)$

$= 7.5$ KN-m

B.M at 'D' $= (8\times3) - \left(4\times1.5\times\left(1.5+\frac{1.5}{2}\right)\right)$

$= 10.5$ KN-m

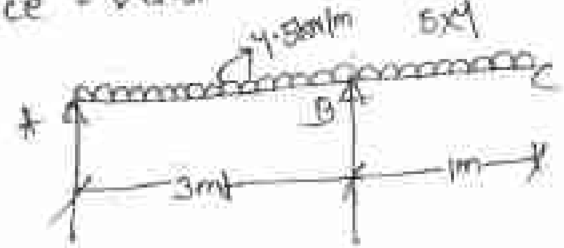
Bending moment at 'E' =

7th sep 2020   2nd period   Monday

Q// A over hanging beam ABC is located as shown in the figure draw S.F & B.M diagram of the loaded beam. and find the point of contraflexure and maximum bending moment.


point of contraflexure the point at which the bending moment diagram changes it sign (+ve to -ve) and vice - version.

Soln




$R_A = 6kN$   $R_B = 12kN$

To find $R_B$

Taking moment at 'A' $= \Sigma m_A = 0$

$R_B \times 3 = 4.5 \times 4 \times \left(\frac{4}{2}\right)$

$\Rightarrow R_B = \frac{36}{3} = 12 kN$

To find $R_A$

T.U.L = T.D.L

$\Rightarrow R_A + R_B = 4.5 \times 4$

$R_A = 18 - 12 = 6 kN$

$\Rightarrow R_A$

$R_A = 6 kN$

Shear force Calculation :-

xox

Shear force at 'A' (Just Left) = 0

Shear force at 'A' (Just Right) = 16 KN

Shear force at 'B' (Just Left) = +6 KN − (4.5×3)

= −7.5 KN

Shear force at 'B' (Just Right) = $(R_A + R_B)$

$- (4.5 \times 3)$

$= 6 + 12 - (4.5 \times 3) = 4.5$ KN

Shear force at 'C' $= (R_A + R_B) - (4.5 \times 4)$

$= (6+12) - (4.5 \times 4)$

$= 0$

Bending moment Calculation.

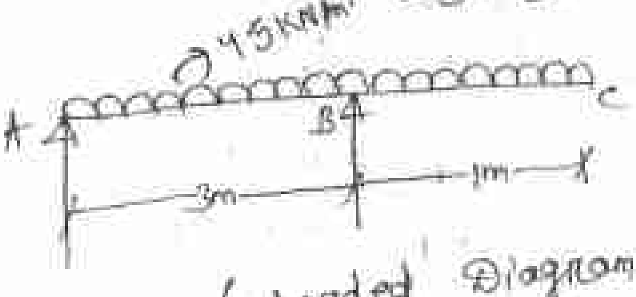
Bending moment at 'A' = 0

Bending moment at 'B' $= (R_A \times 3) - (4.5 \times 3 \times 1.5)$

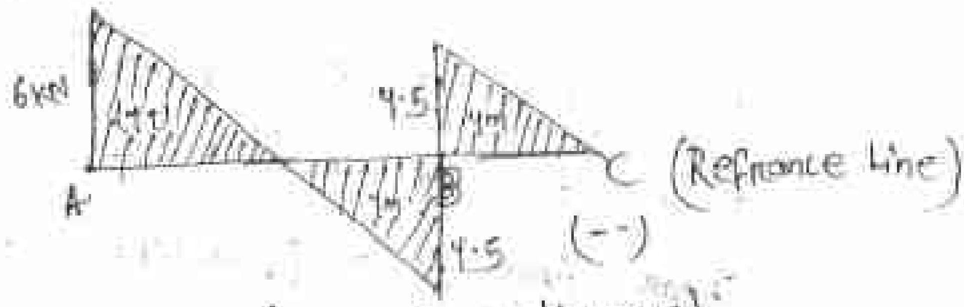
$= -2.25$ KN·m

Bending moment at 'C' $= R_A \times 4 + R_B \times 1 - (4.5 \times 4 \times 2)$

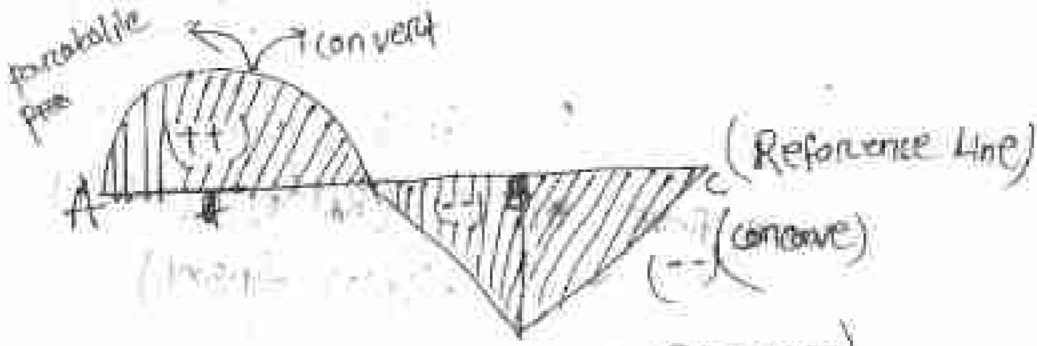
$= 36 - 36 = 0$



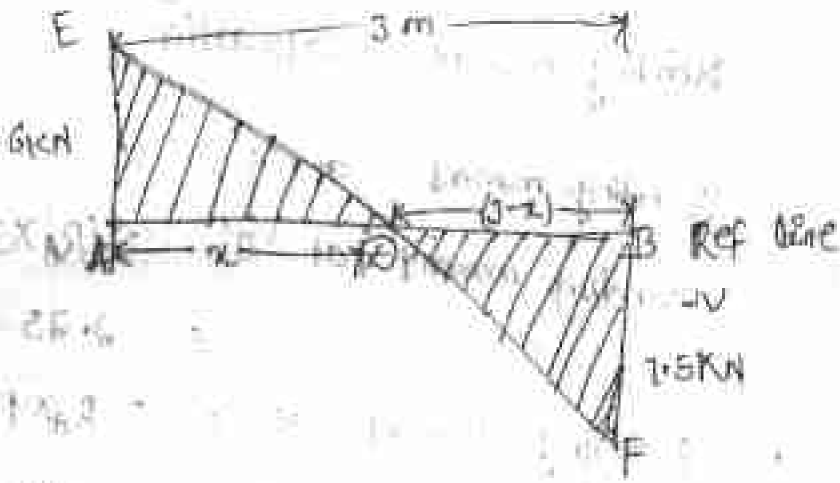

( Loaded Diagram )




(Shear force diagram)




(Bending Moment Diagram)




$\Delta AED \cong \Delta BFD$

$$\frac{x}{3-x} = \frac{6}{7.5}$$

$\Rightarrow 7.5x = 6(3-x)$

$= 7.5x = 18 - 6x$

$7.5x + 6x = 18$

$13.5x = 2.66$

$$x = \frac{18}{13.5} = 1.33$$

$$\Sigma B.M_x = 0$$

$$\Rightarrow R_A \times y - 4.5 \times y \times \frac{y}{2} = 0$$

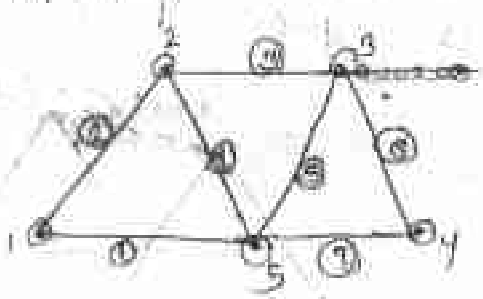
$$\Rightarrow 6 \times y = 4.5 \frac{y^2}{2}$$

$$\Rightarrow y = \frac{12}{4.5} = 2.66$$

unit-8 8 sep 2020 Tuesday

Trusses

* A truss is made up of several bars called members joined together by hinges or rivet.
* But for calculation purposes. The joineds are supposed to be highed or pineped.
* The joints of a truss is called as nodes
* A truss is designed to carry axial loads at ends.




No of member = 7

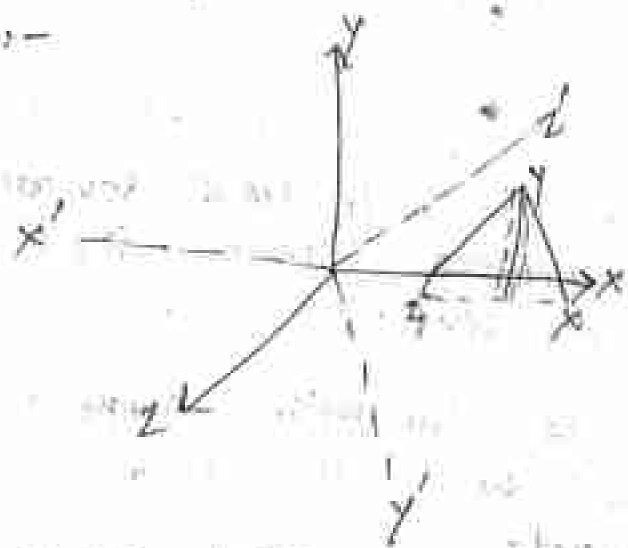
" joints / nodes = 5

Truss

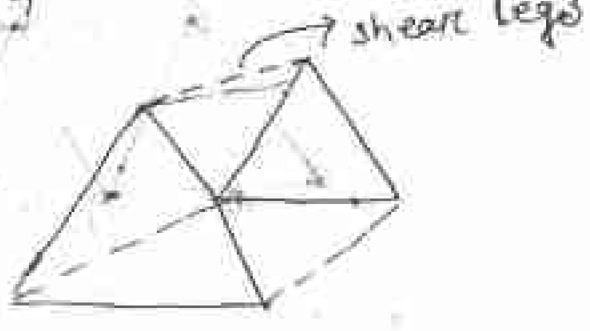
- plane truss
- space truss

**plane truss :-** If the centre line of the members of a truss lie in one plane the truss is known as plane truss.

Ex:-




**space truss :-** If the centre line of a truss don't line in one plane as incase of shear legs in known as space truss.



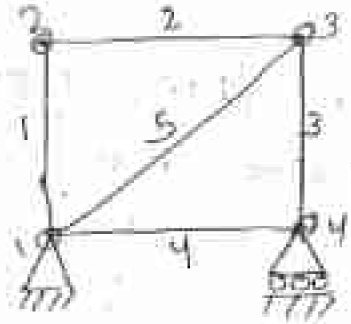

plane truss

- perfect truss ($m = 2J - 3$)
- Imperfect truss ($m \neq 2J - 3$)

(i) Perfect truss :- A truss is said to be imperfect if don't satisfy the eqⁿ

$$\boxed{m \neq 2j-3}$$

Ex:-



$m = 2j - 3$

No of member (m) = 5

" joints (j) = 4

L.H.S (m) = 5

R.H.S $2j-3 = 2\times4-3$
$= 8-3 = 5$

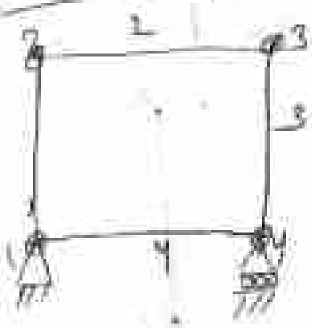
L.H.S = R.H.S

$\boxed{\text{So it is perfect truss}}$.

(ii) Imperfect truss :- A truss is said to be imperfect if don't satisfy the eqⁿ

$$\boxed{m \neq 2j-3}$$

Ex:-



$m = 2J - 3$

$m = 4$

$J = 4$

$L.H.S = 2J - 3$

$= 2 \times 4 - 3$

$= 8 - 3 = 5$

L.H.S ≠ R.H.S | L.H.S ≠ R.H.S

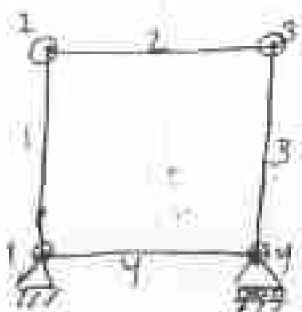
$\boxed{\text{so it is imperfect truss}}$

Imperfect truss

- Deficient truss — $\boxed{m < 2J - 3}$
- Redundant truss — $\boxed{m > 2J - 3}$
  - Internally Redundant truss
  - Externally Redundant truss

i) **Deficient truss:-** If the number of member is less than the required that $m < 2J-3$ that type of truss is called as Deficient truss.

Ex:-



No of member $= 4$

" joint $= 4$

L·H·S $m = 4$

R·H·S $= 2J-3$

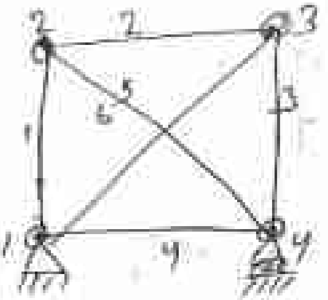
$= 2\times 4 - 3 = 8-3$

$= 5$

(ii) <u>Redundant truss</u> :- If the number of member is more than the required

i.e $m > 2J-3$

Ex:-




No of m $= 6$

No of J $= 4$

L·H·S $= 6$

R·H·S $= 2J-3$

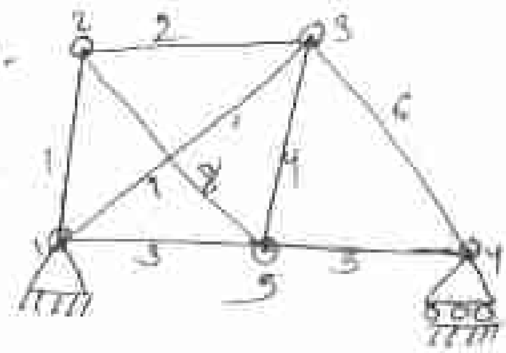
$= 2\times 4 - 3$

$= 5$

$6 > 2J-3$

<u>Internary Redundant truss</u> :- If the number of members is more than the required, i.e $m > 2J-3$, this type of truss is known as internary Redundant truss.

Ex:-




no of member (m) = 8

no of joint (J) = 5

$m \neq 2J-3$

L·H·S $m = 8$

R·H·S $= 2J-3$

$= 2\times5-3$

$= 7$

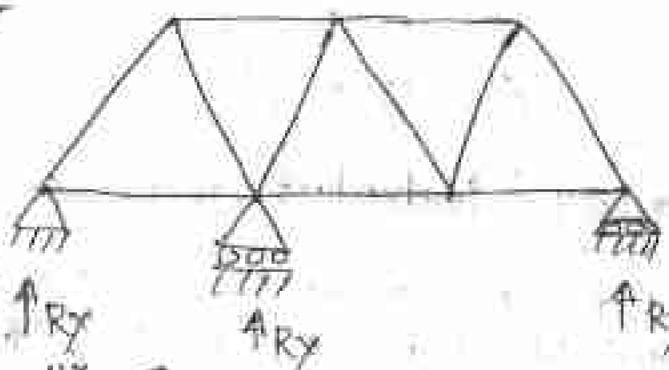
$m > 7$

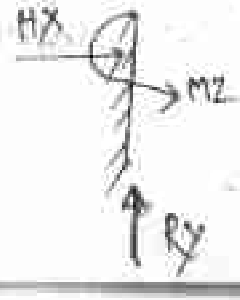
$\boxed{\text{so it is internally redundant truss}}$

Externally Redundant truss :-

If the number of reactions is more than 3, Then The truss is called as externally Redundant truss.

Ex:-




no of unknown relation = 3

9 sep 2020

Here, no of reactions = 2 + 1 + 1 = 4 nos.

4 > 3 so it is an Externally redun-dant truss.

Plane truss

- Perfect truss ($m = 2J - 3$)
- Imperfect truss ($m \neq 2J - 3$)
  - Redundant truss ($m > 2J - 3$)
    - Internally Redundant truss ($m > 2J - 3$)
    - Externally Redundant truss (no. Reaction > 3)
  - Deficient truss ($m < 2J - 3$)

we know that

**In case of 2D**

no of equilibrium eqⁿ = 3 $\left\{ \begin{array}{l} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M_z = 0 \end{array} \right\}$

$\boxed{D = R - 3}$

D = degree of determinancy

**In case of 3D**

no of equilibrium eqⁿ = 6

$\left\{ \begin{array}{l} \Sigma F_{x,y,z} = 0 \\ \Sigma M_{x,y,z} = 0 \end{array} \right\}$

$\boxed{D = R - 6}$

R = no of Reactions

If $D = 0$, $\left[ D = R - 3 = 3 - 3 = 0 \right]$

i.e no of available eq^n = no of unknown reactions.

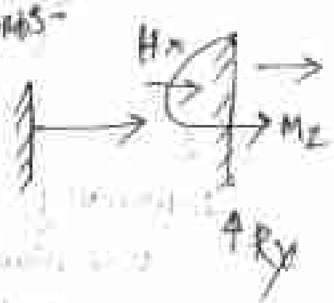
[The structure will be stable and determin'd]

(2) If $D > 0$ ; [The structure will be stable and indetermined]

(3) If $D < 0$, [The structure will be unstable and determinate.]

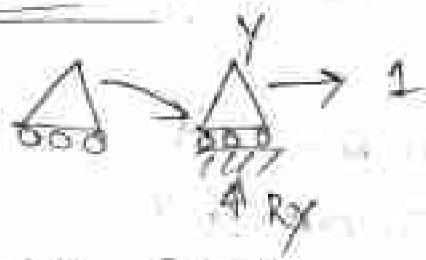
## Supports

(a) fixed supports:-

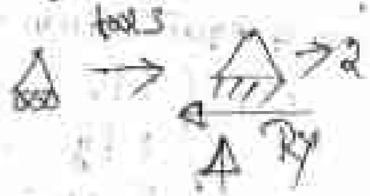



no of unknown Reaction (3)

(b) Roller support:-




(c) Hinged support




## Assumptions to solve plane truss:—

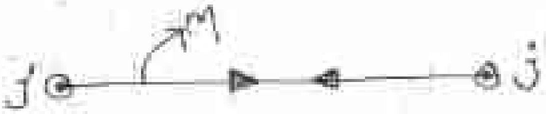
→ All the members are in one plane

→ The members are connected at their ends by smooth hinges.

> The ~~loads~~ are applied at their ~~point~~ only Joint.

> The members are weightless.

> The forces in the member are only axial so that the members are in either tension or compression.



Tension - Away from joint

M: compression

<u>10 sep 2020</u>

## Degree of indeterminacy (Truss)

Dox

- Static Indeterminacy
- Kinematic Indeterminacy

① <u>Static Indeterminacy:-</u>

It is related to the unknown forces in the given structure.

② <u>Kinematic Indeterminacy:-</u>

It is related to the available unknown degree of freedom.

<u>Static Indeterminacy</u>

- External Static Indeterminacy ($D_{se}$)
- Internal static Indeterminacy ($D_{si}$)

→ It is related to unknown external support reaction.

→ It is related to unknown Internal reaction of member forces.

Total static indeterminacy ($D_s$) = $D_{se} + D_{si}$

**Imp** Static Indeterminacy :- The structure is called statically determinate if it can be analised by using equation of equilibrium.

| Eq$^n$ of equilibrium in 2D | Eq$^n$ of equilibrium in 3D |
|---|---|
| (i) $\Sigma F_x = 0$ | (i) $\Sigma F_x = 0$ |
| (ii) $\Sigma F_y = 0$ | (ii) $\Sigma F_y = 0$ |
| (iii) $\Sigma M_z = 0$ | (iii) $\Sigma F_z = 0$ |
| | (iv) $\Sigma M_x = 0$ |
| | (v) $\Sigma M_y = 0$ |
| | (vi) $\Sigma M_z = 0$ |
| The no of equation of equilibrium is '3' in case of 2D | The no of eq$^n$ of equilibrium is '6' in case of 3D. |

(i) If the given structures members cannot be analised by using equilibrium equations. Then they are called as statically Indeterminate structure or Redundant structure.

$$\left\{\begin{array}{ll} 2x + 3y + f = 0 & 2x + 3y + 5z = 0 \\ 3x + 4y + 8 = 0 & 4x + 6y + 8z = 0 \end{array}\right\}$$

$\boxed{\text{static indeterminacy} = \text{unknown} - \text{known}}$

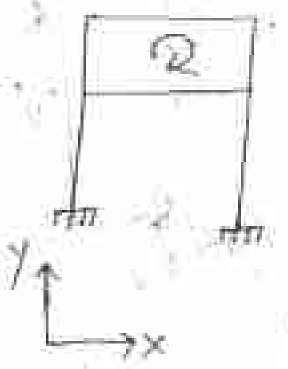
Total static Indeterminacy ($D_s$) = $D_{se} + D_{si}$

External static Indeterminacy :-

$(D_{se}) = r_e - 3$ [no of equilibrium ← 3 equation; $r_e$ : external reaction]

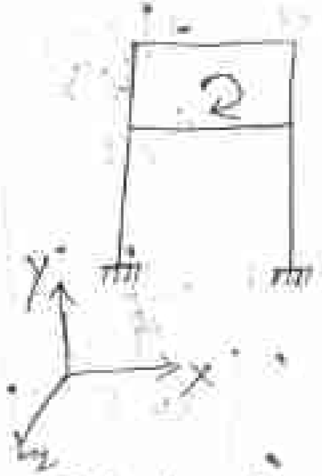
## Internal static Indeterminacy :-

xox ——— xox



$D_{si} = 3C - r_e$

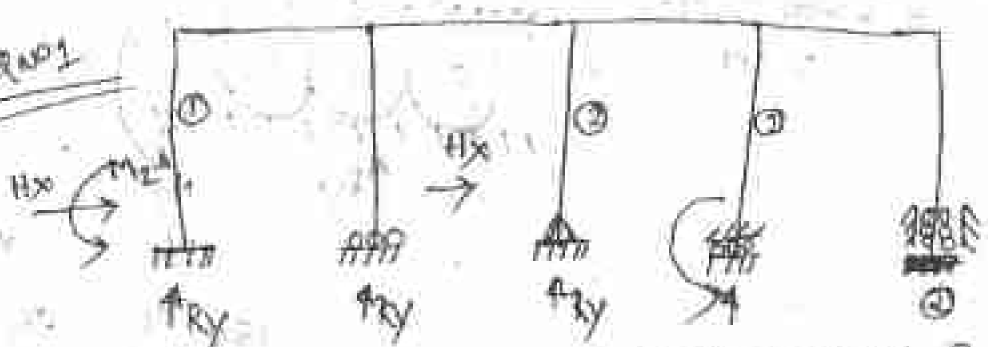
C → no of closed loops.



$D_{si} = 6C - r_e$

$r_e$ →

Q.no 1



Find out degree of Indeterminacy ?

Sol^n = $D_s = D_{se} + D_{si}$

$= (3+1+2+2+2) - 3$

$= 10 - 3 = 7$

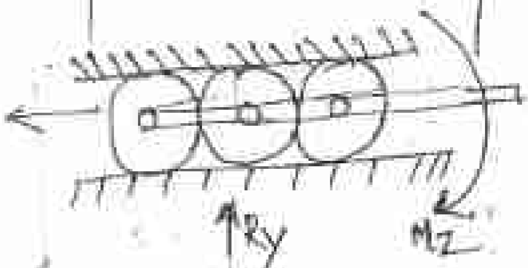
11 sep 2020 4th period

## Types of support and their reactions :-

# Types of supports and their reactions

| Types of support | At the end | At the | External reaction in 2D | External reaction in 3D |
|---|---|---|---|---|
| ① Fixed support | Rx, Mz (or) Ry | – | ③ Rx, Ry, Mz | ⑥ Rx, Ry, Rz, Mz, My, Mx |
| ② Hinged support | Rx, Ry (or) Ry | | ② Rx, Ry | ③ Rx, Ry, Rz |
| ③ Roller support | Ry | | ① → Ry | ① → Ry |

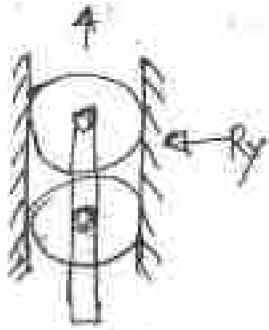
④ Horizontal Guided Roller




2D: ② Mz, Ry

3D: ② Mz, Ry

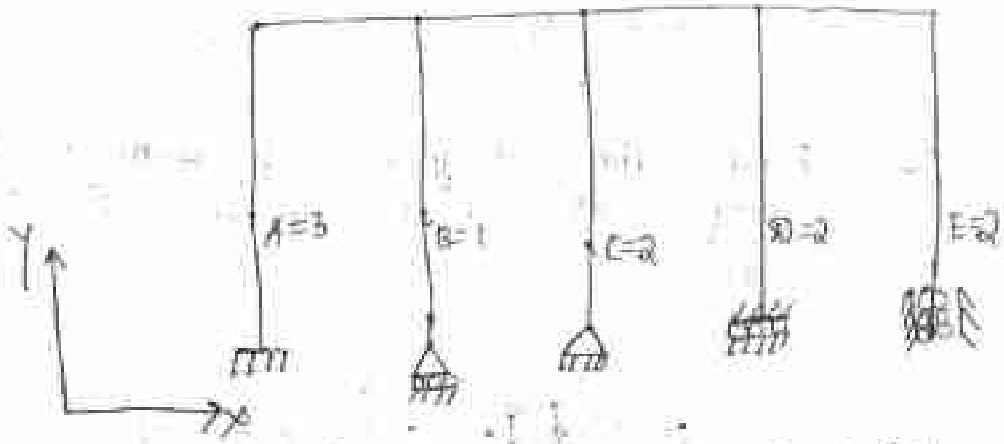
⑤ Vertical Guided Roller




2D: ② Rx, Mz

3D: ② Rx, Mz

Q1 Find the degree of static indeterminacy of given frame as shown in the figure below.




Sol Total static Indeterminacy $D_s = D_{se} + D_{si}$

External static indeterminacy - $D_{se} = r_e - 3$

$= 10 - 3 = 7$

Internal static Indeterminacy - $D_{si} = 3C - r_r$

$C \rightarrow$ No of closed loops

$r_r \rightarrow$ Released Reaction

$D_{si} = 3C - r_r \; (C = 0, r_r = 0)$

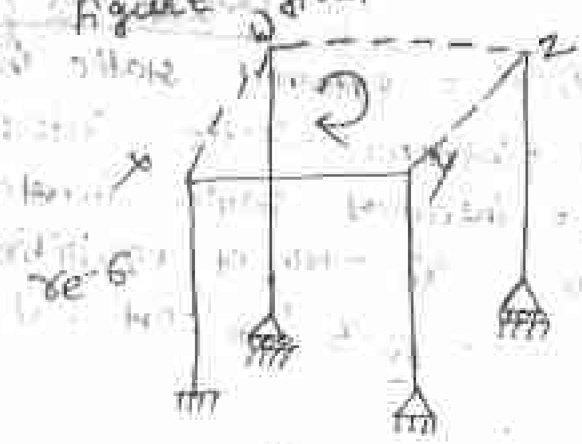
$= 3 \times 0 - 0 = 0$

Total static Indeterminacy $D_s = D_{se} + D_{si}$

$= 7 + 0 = 7$

Degree of static indeterminacy is '7'

Q2 Find the degree of static indeterminacy of the figure given below.




$D_s = D_{se} + D_{si}$

$D_{se} = r_e - 6$

$= (6 + 3 + 1 + 1) - 6$

$= 11 - 6 = 5$

$D_{si} = 6C - r_r$

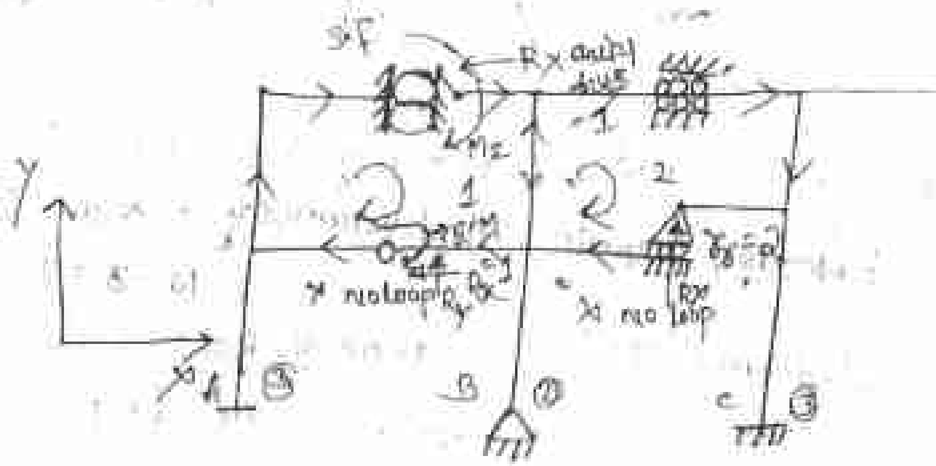
$C = '1'$

$D_{si} = 3\times1 = 3$

$D_{S} = 5+3 = 8$

Degree of indetermancy = '11'

Q3 Find the degree of static indeterm-ancy as shown in the figure below.




12 Sep 2020

~~$D_{se} = (3+1+3) - 3$~~

~~$8-3=5$~~

~~$D_{si} = 3c$~~

~~c → No of Closed loops~~

~~released reaction~~

Total static indetermancy

$$\boxed{D_s = D_{se} + D_{si}}$$

$D_{se}$ = External static indetermancy

$D_{si}$ = internal static indetermancy

~~$D_{si}$ = internal static indetermancy~~

$D_{se}$ = $r_e$ − No. of equilibrium equation

$= r_e - 3$ [$r_e$ - external reaction

$D_{se} = (3+2+3) - 3$

$= 8 - 3 = 5$

$D_{si} = 3c - r_r$

c → no of closed loops

$r_e$ → released reactions

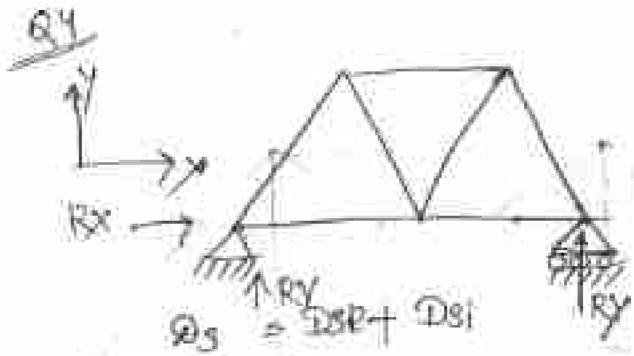
The means $c = 2$, $r_r = 5$

$D_{si} = 3 \times 2 - 5 = 6 - 5 = 1$

$D_s = D_{se} + D_{si} = 5 + 1 = 6$

The structure is degree of indeterminacy = '6'

Static ~~I~~ indetermancy in pin jointed structure
Degree of




$D_s = D_{se} + D_{si}$

$D_{se} = r_e - 3$

$\boxed{D_{si} = m - 2J + 3}$

m → no of members.

J → no of joints.

Sol$^n$ - $D_s = D_{se} + D_{si}$

$D_{se} = r_e - 3$

$r_e = 2 + 1 = 3$

$D_{se} = 3 - 3 = 0$ The truss is externally determinate.

$D_{si} = m - 2J + 3$

$m = 7 \qquad J = 5$

$Dsi = 7 - (2 \times 5) + 3$

$= 7 - 10 + 3$

$= -3 + 3 = 0$ (The truss is internally determinate)

Total static Indeterminacy

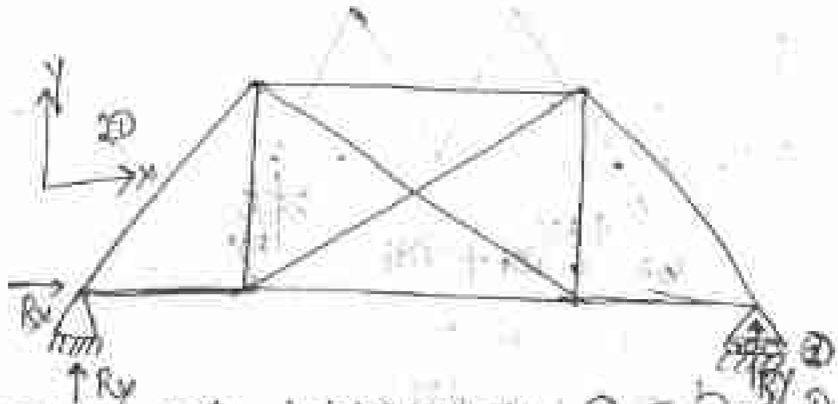
$= Dse + Dsi$

$= 0 + 0 = 0$

That means the truss is statically determinate.

$$\left\{ \begin{array}{l} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M_z = 0 \end{array} \right\}$$

**Q5** Find out the degree of indeterminacy of the given truss as shown in the figure below.




**Sol** Total static indeterminacy $D_S = Dse + Dsi$

$Dse = Re - 3$

$= (3 - 3) = 0$

The truss is Externally determinate

$Dsi = \cancel{m + 2j}$

$m - 2j + 3$

$m = 10 \quad j = 6$

$= \cancel{...} + 3 \quad 10 - (2 \times 6) + 3$

$= \cancel{0} + 1$

The truss is internally indeterminate.

$D_s = D_{se} + D_{si}$

$= 0 + 1$

$= 1$

The truss is degree of indetermancy of '1'.

<u>Analysis truss</u> :- The analysis of truss is don by the following methods.

1. Analytical method → method of joints / → method of sections
2. Graphical method

① <u>Method of joints</u> :-

<u>procedure</u> ① Determine the support reactions in case of simple supported truss.

→ consider any joint with minimum unknown members meeting at that joint is not greater than 2

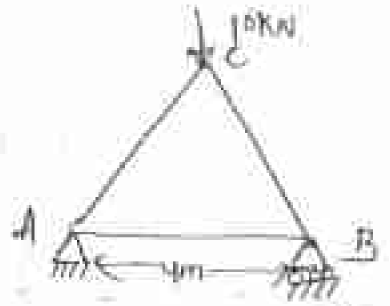
→ Assume all the forces in the members are tinsile in nature. But if after calculation the value of member forces comes -ve since it is compressive in nature. and the -ve value is to put the next calculation of member forces.

→ use equilibrium equations or conditions to get the unknown member forces.

$$\begin{bmatrix} \Sigma F_x = 0 \\ \Sigma F_y = 0 \end{bmatrix}$$

Repeat for other joints.

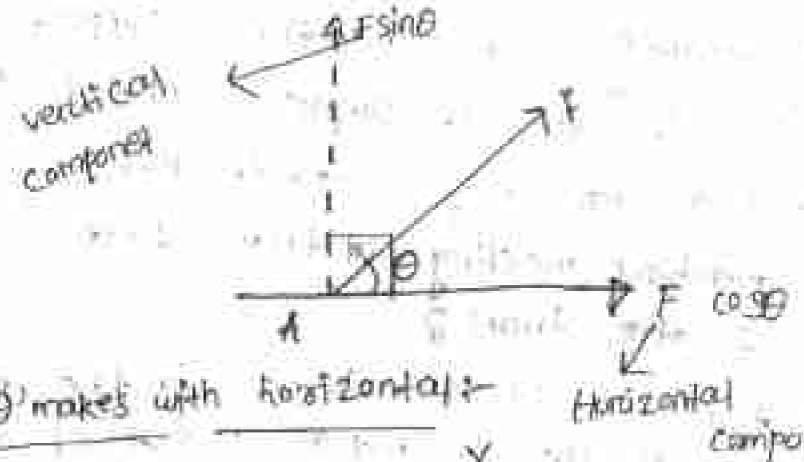
Q1 Find the forces in the members 'AC' 'AB' BC and 'BC' by using method of joints of the given truss as shown in figure.



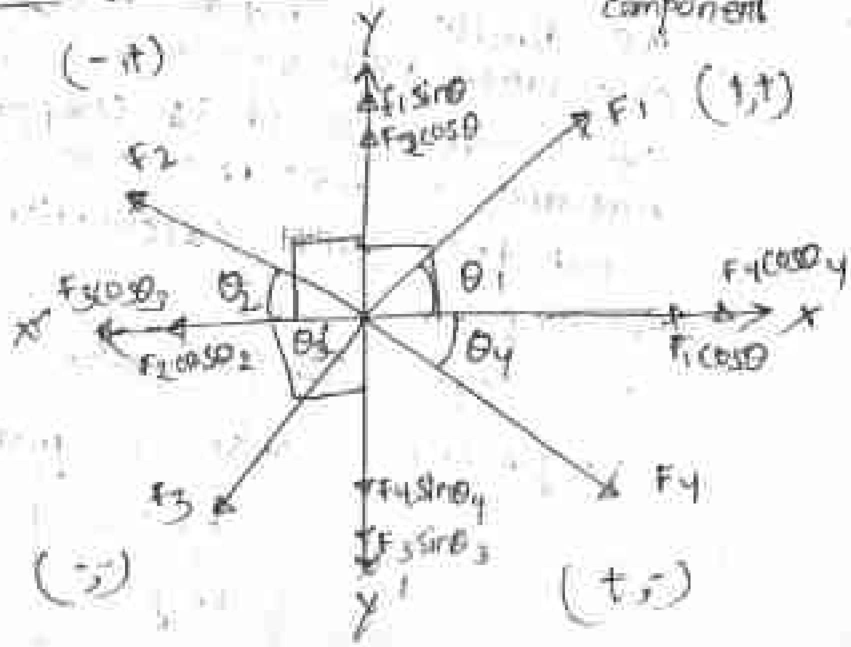

we know that 'A' inclined force has two components.

(i) Horizontal component

(ii) vertical component




'θ' makes with horizontal:-

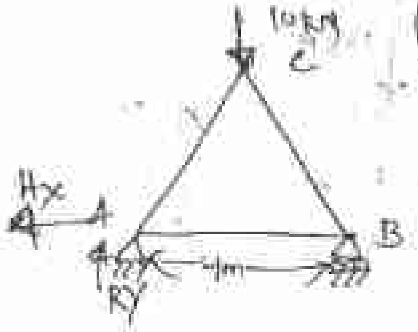


~~15 Sept 2020~~ 17 Sep 2020

~~3-~~

~~Long~~ :

Q1 Find out the member forces in the given truss.




(by using method of joints)

Sol$^n$ :-

Step - I

statically indeterminate

$$D_s = D_{si} + D_{se}$$

$$D_{se} = r_e - 3$$

$$D_{si} = m - 2J + 3$$

$$D_{se} = r_e - 3$$

$$r_e = 3$$

$$D_{se} = r_e - 3 = 3 - 3 = 0$$

That means the truss is externally determinate.

$$D_{si} = m - 2J + 3$$

$$= 3 - 2 \times 3 + 3 = 3 - 6 + 3 = 0$$

The truss internally determinate.

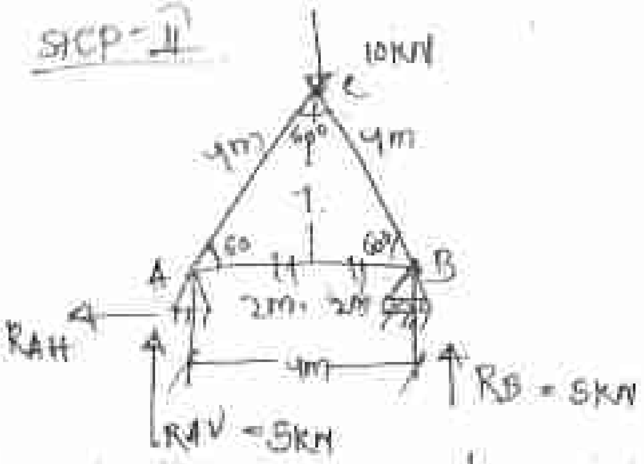
$$D_s = D_{se} + D_{si}$$

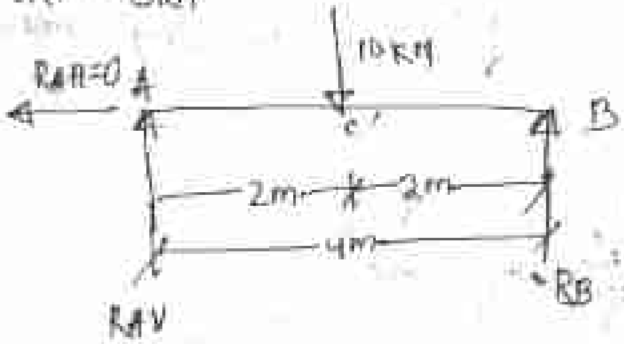
$$= 0 + 0$$

$$= 0$$

So the truss is statically determinate.

Step - II




Taking moment at 'A'

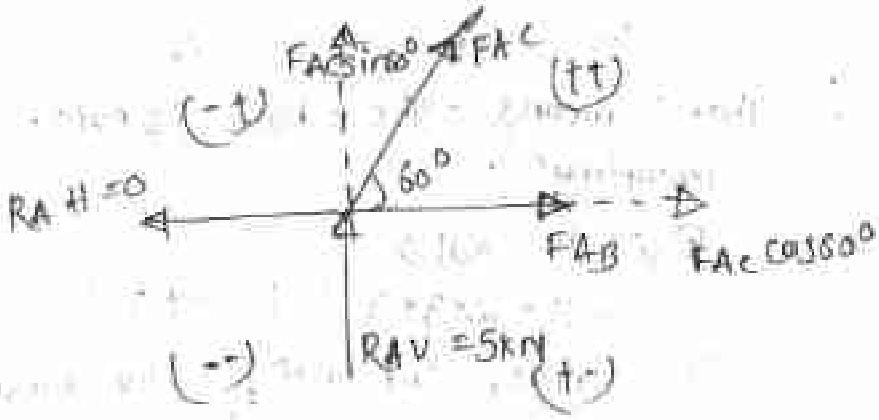
$T \cdot A \cdot M = T \cdot C \cdot M$

$\Rightarrow R_B \times 4 = 10 \times 2$

$\Rightarrow R_B = \frac{20}{4} = 5$

Step = III

Consider joint (A)




$\Sigma f_x = 0$

$F_{AB} + F_{AC} \cos 60^\circ - R_{AH} = 0$

$F_{AB} + F_{AC} \cos 60^\circ = 0$ ——— (1) eqn

$\Sigma F_y = 0$

$F_{AC} \sin 60^\circ + R_{AV} = 0$

$\Rightarrow F_{AC} = -5/\sin 60^\circ = -1.776\ kN$.

$\Rightarrow F_{AC} = -1.667\ kN$

$= 1.667\ kN$ (compressive)

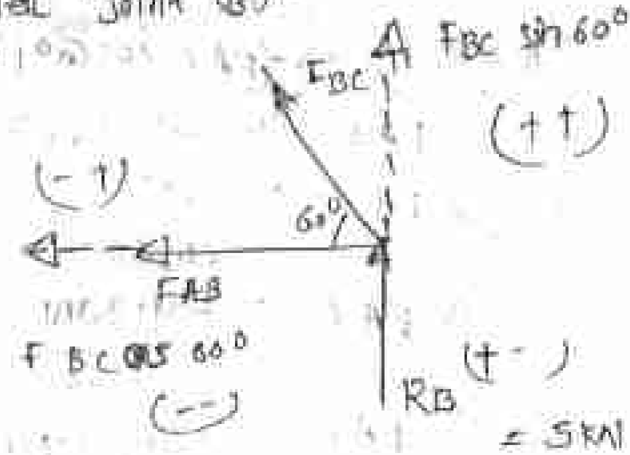
put the value of $F_{AC}$ in the eqn (1)

$F_{AB} + F_{AC} \cos 60^\circ = 0$

$\Rightarrow F_{AB} - 1.667 \times \cos 60^\circ =$

$F_{AB} =$

Consider joint (B)




$\Sigma F_x = 0$, $\Sigma F_y = 0$

$-F_{BC} \cos 60^\circ - F_{AB} = 0$

$\Rightarrow -(F_{BC} \cos 60^\circ + F_{AB}) = 0$

$\Rightarrow F_{BC} \cos 60^\circ + F_{AB} = 0$

$\Rightarrow F_{BC} = \dfrac{-F_{AB}}{\cos 60^\circ}$

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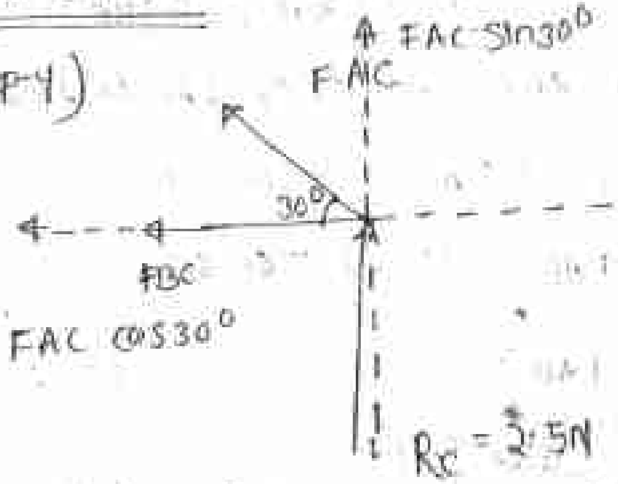
(step 4)




$\Sigma H = 0$ , $\Rightarrow -(FAC \cos 60^\circ + FBC) = 0$

$\Rightarrow FAC \cos 60^\circ + FAB = 0$

$\Rightarrow FAC = -\frac{FBC}{\cos 60^\circ}$

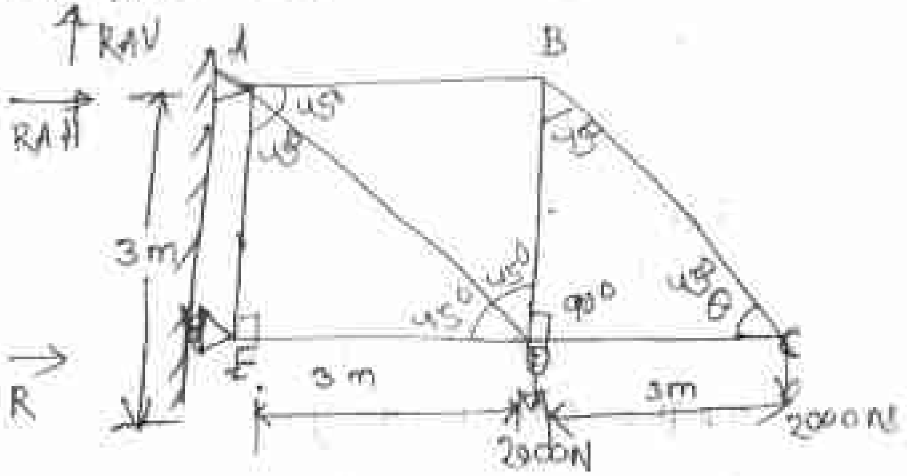
$\Rightarrow FAC = \frac{-4.33N}{\cos 60^\circ}$

$\Rightarrow FAC = -5N$ (Tensile)

$= 5N$ (compressive)

| Force in members | Magnitude | Nature |
|---|---|---|
| FAC | 5 N | Compressive |
| FBC | 4.33N | Tensile |
| FBC | 8.66N | compressive |

Q2 Determine the force in all the members of the given truss.




Sol$^n$ $\tan\theta = \frac{BD}{DC}$

$\Rightarrow \tan\theta = \frac{3}{3}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{3}{3}\right)$

$\Rightarrow \theta = \tan^{-1}(1) = 45^0$

Step - 1 Total static indeterminancy $D_s = D_{se} + D_{si}$

External static indeterminancy

$D_{se} = r_e - 3$

$r_e$ = no of unknown reactions

$D_{se} = 3 - 3 = 0$

So the truss is externally determinancy.

Internal static indeterminancy

$D_{si} = m - 2j + 3$

m → no of members

j → no of joints

$\left(\begin{array}{l} m = 7 \\ j = 5 \end{array}\right)$

$D_{si} = 7 - 2\times5 + 3 = 0$

So the truss is internally determinate

$D_s = D_{se} + D_{si}$

$= 0 + 0 = 0$

So the truss can be solved by the conditions of equilibrium.

Taking moment at A : $\Sigma M_A = 0$

T.A.M = T.C.M

$\Rightarrow R_e \times 3 = 2000 \times 3 + 2000 \times 6$

$\Rightarrow R_e = 6000\,N (\rightarrow)$

$R_e + R_{AH} = 0$

$R_{AH} = -R_e = -6000\,N (\rightarrow)$

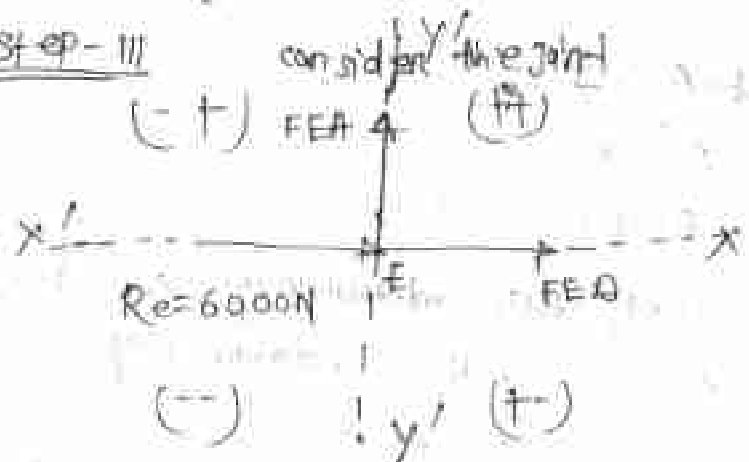
$= 6000\,N (\leftarrow)$

$R_{AH} = 6000\,N (\leftarrow)$

$R_{AV} = 4000\,N (\uparrow)$

R

Step - III    consider the joint




$\Sigma H = 0$ , $\Sigma V = 0$    $\Sigma H = 0$

$R_e + F_{ED} = 0$

$\Rightarrow 6000\,N + F_{ED} = 0$

$\Rightarrow F_{ED} = -6000\,N$

$\Rightarrow F_{ED} = 6000\,N$ (Comp)

$\Sigma V = 0$

$F_{EA} = 0$

$F_{EA} \Rightarrow$

zero/null force member.

23 Sep 2020

Method of section

procedure to solve a truss by method of section :—

**step - 1** Find support reactions if required.

**step - II** Identify the member and highlight it in FBD of truss.

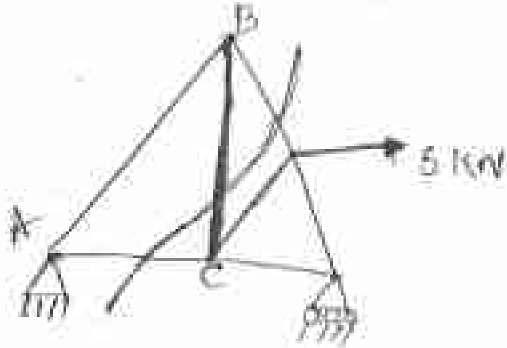
**step - III** Divide the entire truss into two parts but not cut more than 3 members at a time.

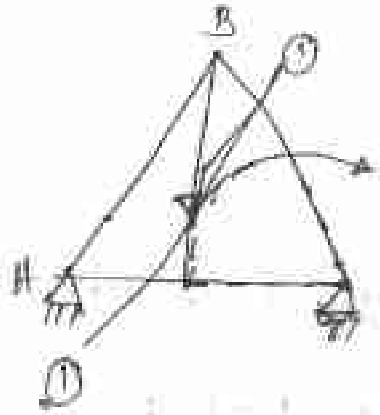
**step - iv** Mark the consider portion with line and other part in dotted line.

**step - V** Assume all the forces are tensile nature.

**step - VI** Apply eqn of static equilibrium

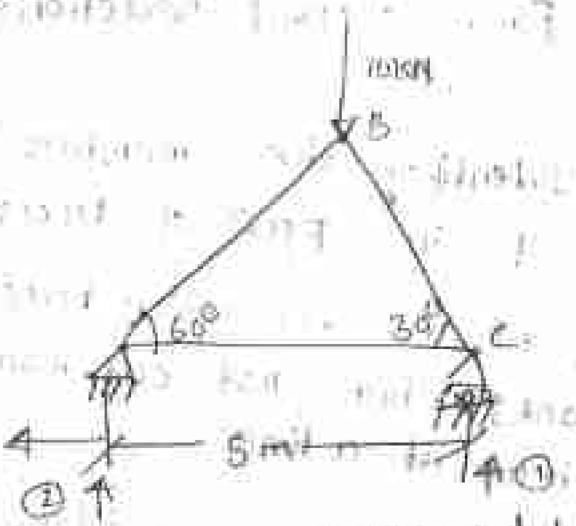
$$\begin{bmatrix} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma F_z = 0 \end{bmatrix}$$




29 Sep 2020

**Q)** Find the forces in the members AB, BC & CA of the truss in the figure




**Step-I** Total static indeterminacy

$D_s = D_{se} + D_{si}$

External static indeterminacy

$D_{se} = r_e - 3$

$= 2 + 1 - 3$

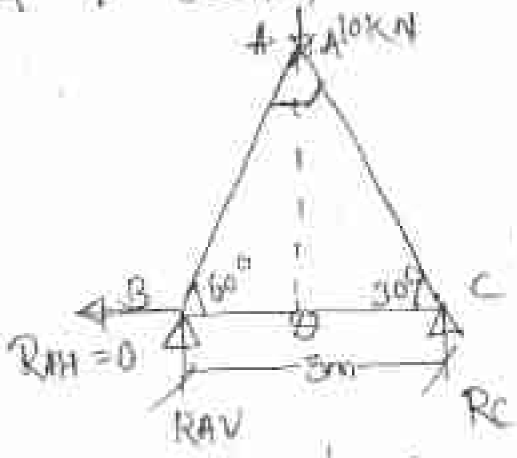
$= 3 - 3 = 0$

The truss is internally determinate

$D_s = D_{se} + D_{si} = 0 + 0 = 0$

So the truss is determinate.

Method of section

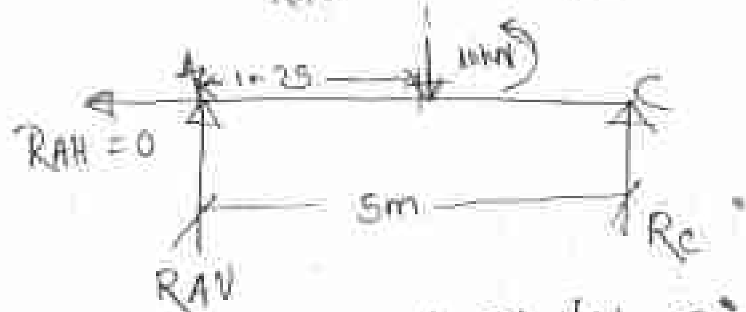



$AB = 5 \times \cos 60^\circ = 2.5$

$BC = 5 \times \sin 60^\circ = 4.33$

$AD = 2.5 \times \cos 60^\circ = 1.25$

$BD = 2.5 \times \sin 60^\circ = 2.16$




Taking moment at 'A' $\Sigma M_A = 0$

$ACW \cdot M = T \cdot C \cdot M$

$\Rightarrow R_C \times 5 = 10 \times 1.25$

$\Rightarrow R_C = \frac{10 \times 1.25}{5}$

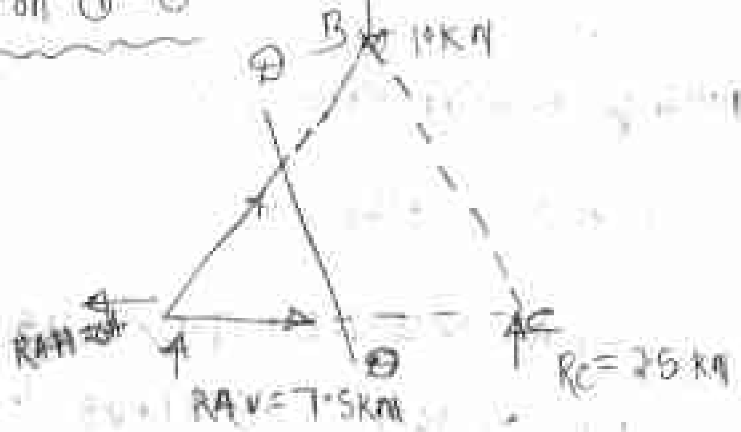
$= R_C = 2.5\,KN$

$T \cdot U \cdot L = T \cdot D \cdot L$

$\Rightarrow RAV + R_C = 10KN$

$\Rightarrow RAV = 10 - R_C = 10 - 2.5 = 7.5\,KN$

Section ①-①



Consider section ① - ①

$\Sigma M_c = 0$

$F_{AB} \times 4.330 + R_{AV} \times 5 = 0$

$\Rightarrow R_c = 2.5 kN$

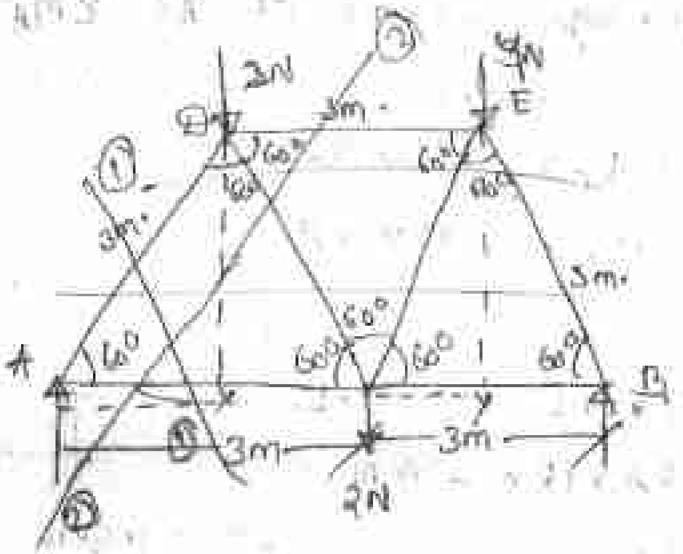
$F_{AB} = \dfrac{-R_{AV} \times 5}{4.330} = \dfrac{-7.5 \times 5}{4.330}$

$= -8.66$

$= -8.660 \, kN$

$= 8.660 \, kN$ (Compressive)

Q2 Find the forces in the members of the given truss.




Step 1

Taking moment at 'B', $\Sigma M_B = 0$

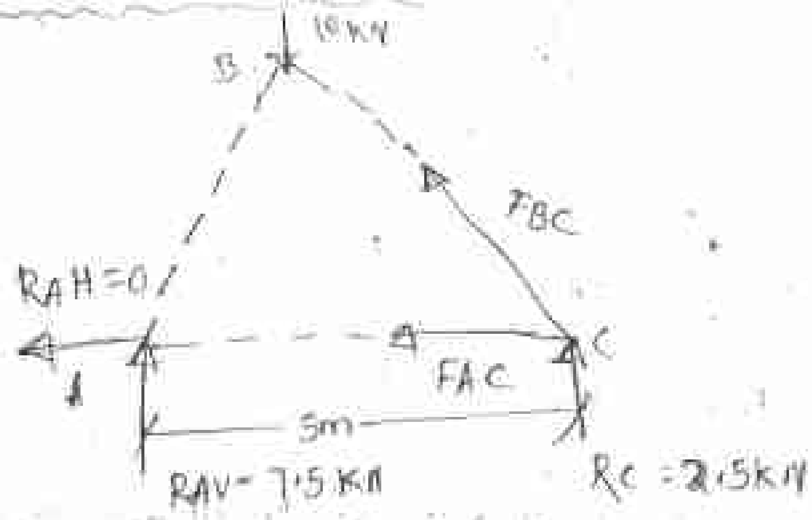
$-F_{AC} \times B② + R_{AV} \times A② = 0$

$\Rightarrow -F_{AC} \times 2.165 + 7.5 \times 1.25 = 0$

$\Rightarrow F_{AC} \times 2.165 = 7.5 \times 1.25$

$F_{AC} = \dfrac{7.5 \times 1.25}{2.165} = 4.330 \, kN$ (Tensile)

Consider section ② · ②




Taking moment at 'A' $\Sigma M_A = 0$

$\Rightarrow -R_C \times 5 - F_{BC} \times AC = 0$

$\Rightarrow -(R_C \times 5 + F_{BC} \times AB) = 0$

$\Rightarrow (R_C \times 5 + F_{BC} \times 2.5) = 0$

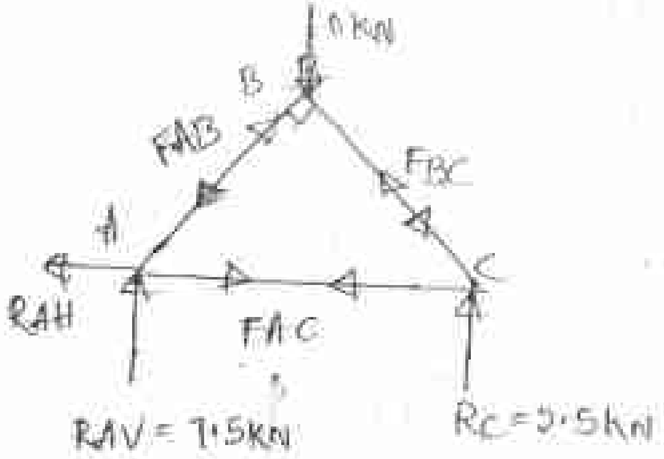
$\Rightarrow F_{BC} \times 2.5 = -R_C \times 5$

$\Rightarrow F_{BC} = \dfrac{-(2.5 \times 5)}{2.5} = -5\,KN$

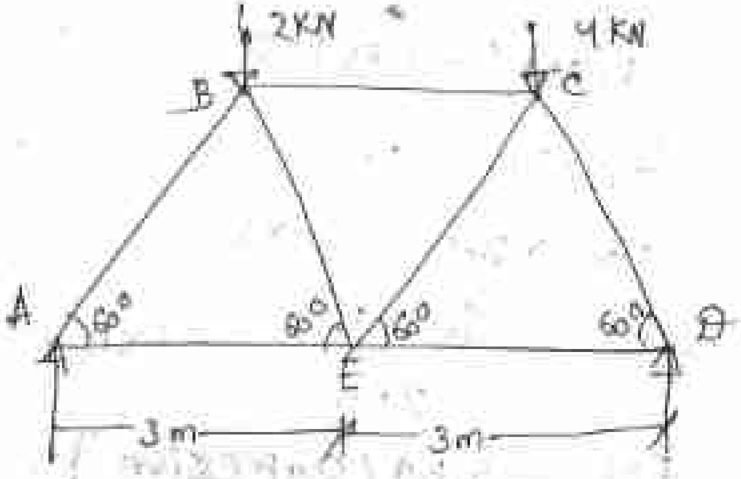
$F_{BC} = 5\,KN$ (Compressive)

| SL no | forces in the members | Magnitude | nature of force |
|---|---|---|---|
| 1 | FAB | 8.66 KN | Compressive. |
| 2 | FAC | 4.33 KN | Tensile |
| 3 | FBC | 5 KN | Compressive |

k)




Imp Q2 Find the forces in all the members of the girder (beam) indicating whether the force is compressive or tensile.




(By using method of sections)

$m\angle x = 90^\circ$

$Ax = 3\cos 60^\circ$
$= 1.5\,mt$

$Bx = 3\sin 60^\circ$
$= 2.598\,mt$

$Ey = 3\cos 60^\circ$
$= 1.5\,mt$

$Cy = 3\sin 60^\circ$
$= 2.598\,mt$

Step-1

Soln Total static Indeterminacy

$D_s = D_{se} + D_{si}$

External static indeterminacy

$D_{se} = r_e - 3$

$= 2$

Internal static Diterminacy,

$D_{si} = m - 2j + 3$

$= 7 - 2\times 5 + 3$

$= 7 - 10 + 3 = 0$

$D_S = D_{se} + D_{si} = 0 + 0 = 0$

The truss can be determined

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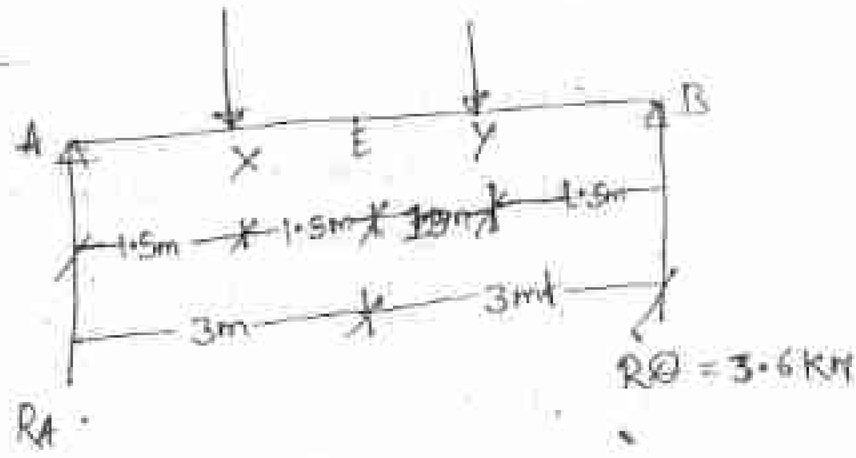
Step - II




To find RB, $\Sigma M_A = 0$

T.AM = T.C.M

$\Rightarrow R_B \times 6 = 2 \times 1.5 + 4 \times 4.5$

$R_B \times 6 = 2 \times 1.5 + 4 \times 4.5$

$R_B = 3.6$ KN

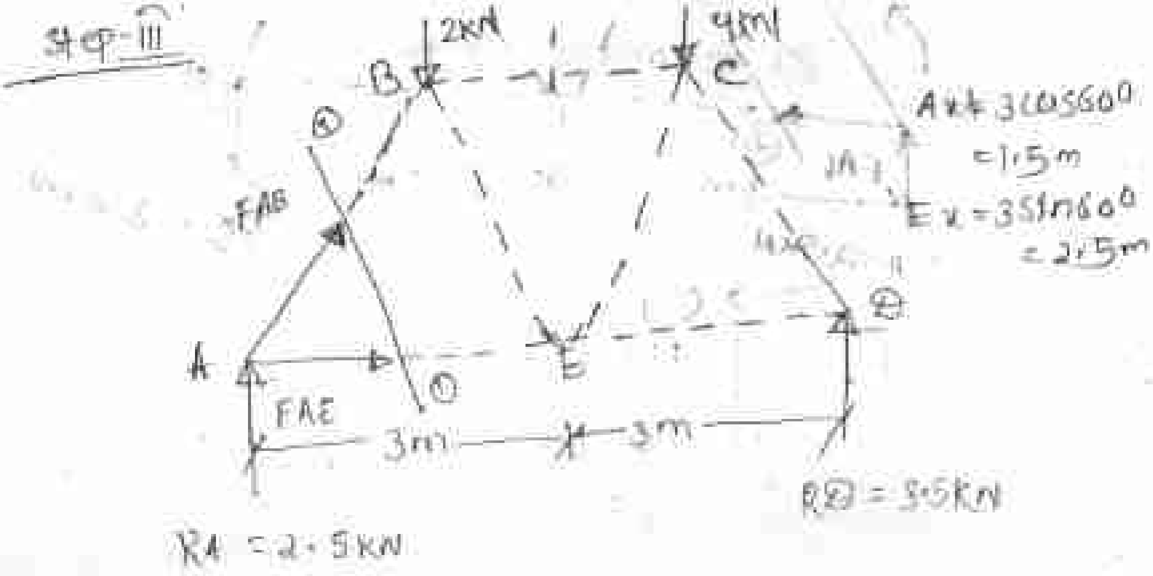
To find RA

T.U.L = T.D.L

$\Rightarrow R_A + R_B = 2 + 4 = 6$

$R_A = 6 - 3.5 = 2.5$ KN

Step - III



Consider section ①-①

To find FAB $\Sigma M_E = 0$

$\Rightarrow R_A \times 3 + FAB \times 3\sin 60^\circ = 0$

$\Rightarrow 2.5 \times 3 + FAB \times 2.598 = 0$

$\Rightarrow FAB \times 2.598 = -2.5 \times 3$

$\Rightarrow FAB = \dfrac{-(2.5 \times 3)}{2.598} = -2.886$ kN

FAB = 2.886 kN (Compressive)

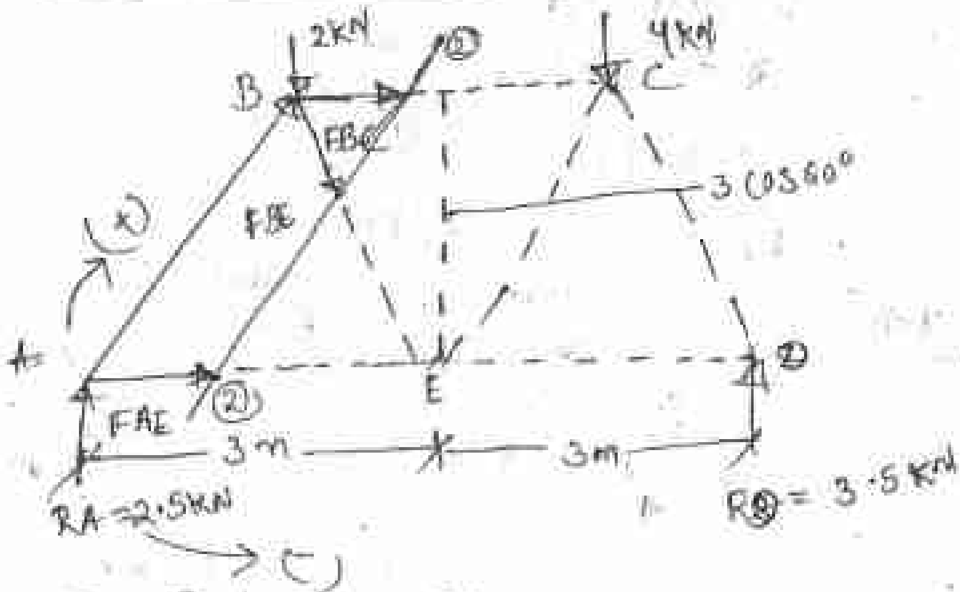
To find FAE, $\Sigma M_B = 0$

$R_A \times 3\cos 60^\circ - FAE \times 3\sin 60^\circ = 0$

$\Rightarrow FAE = \dfrac{2.5 \times 3\cos 60^\circ}{3\sin 60^\circ}$

$= \dfrac{2.5 \times 1.5}{2.598} = 1.330$ kN (Tensile)

Consider section 2-2




To find FBC, $\Sigma M_E = 0$

$\Rightarrow FBC \times 3\sin 60^\circ + (2.5 \times 3) - 2 \times 3\cos 60^\circ = 0$

$\Rightarrow FBC = -1.732$ kN

$FBC = 1.732$ KN (compressive)

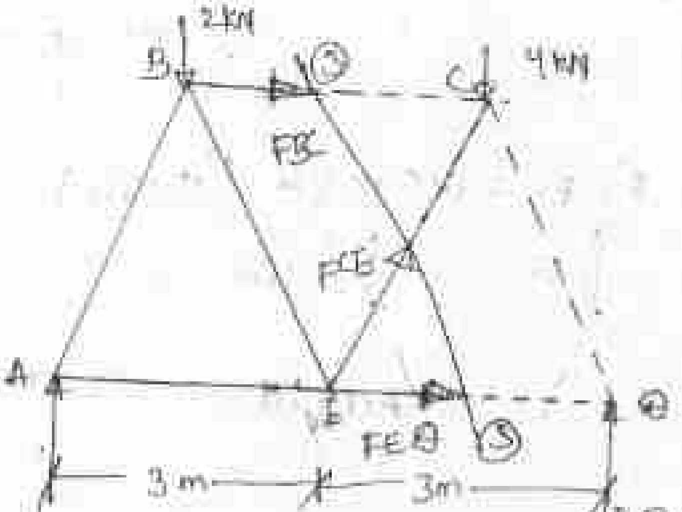
To find FBE $\Sigma M_c = 0$

$= -FBE \times 3\sin 60^0 - (2\times3) - FAE \times 3\sin 60^0 + 2.5\times4.5 = 0$

$\Rightarrow FBE = \frac{-6}{2.598} = -2.300$ KN

$FBE = 2.300$ KN (Tensile)

<u>29 sep 2020</u>




RA = 2.5KN

RD = 3.5KN

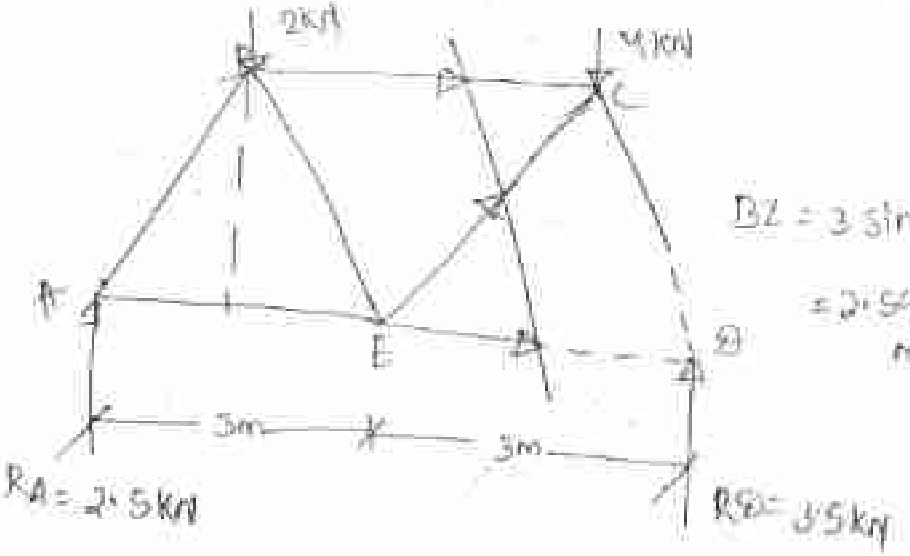
Consider section ③-③

Let us consider the left portion of the truss of the section ③-③

To find FED, $\Sigma M_c = 0$

$\Rightarrow -(2\times3) - (FED \times 2.598) + (2.5+4.5) = 0$

$= FED = 2.020$ (Tensile)



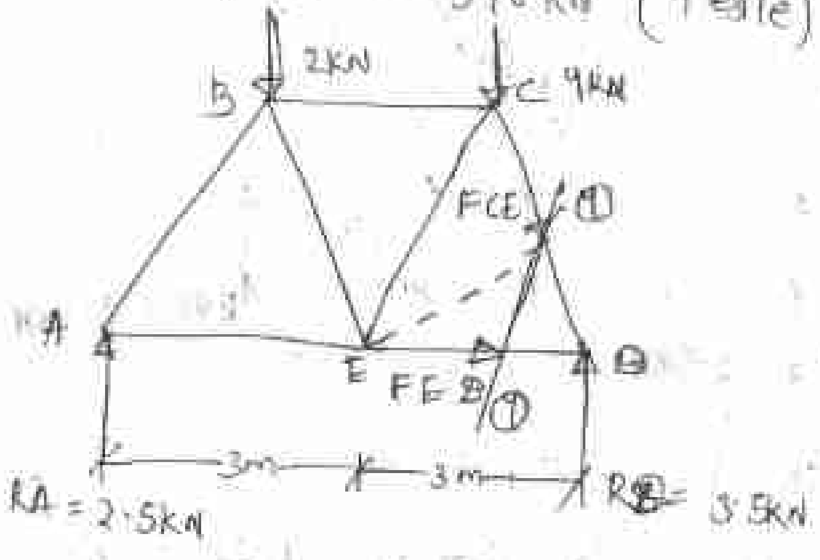

BZ = 3 sin 60°

= 2.598 m.

Taking moment at 'B', $\Sigma M_B = 0$

$\Rightarrow -FED \times 2.598 + RA \times 1.5 - FCE \times 2.598 = 0$

$FCE = 0.576$ KN (Tensile)




To find FCE, $\Sigma M_E = 0$

$FCE \times 3\sin 60° + 4 \times 1.5 + 2.5 \times 3 - 2 \times 1.5 = 0$

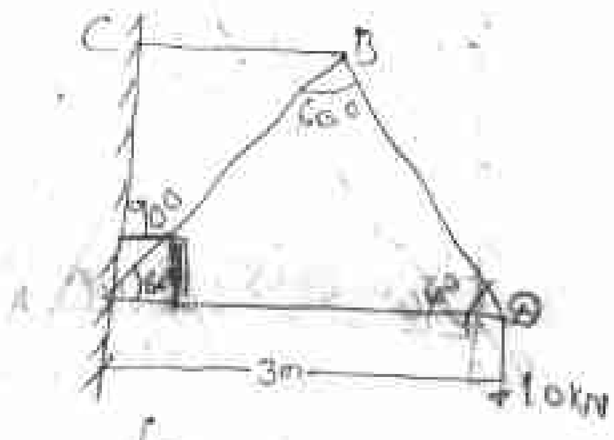
$FCE = 13.09$

598

| SL NO | Forces in the member | Magnitude | Nature |
|---|---|---|---|
| (1) | FAB | 2.886 KN | Compressive |
| (2) | FAE | 4.330 KN | Tensile |
| (4) | FBE | 2.300 KN | Tensile |
| (3) | FBC | 1.732 KN | Compressive |
| (5) | FED | 2.020 KN | Tensile |
| (6) | FCE | 0.576 KN | Tensile |
| (7) | FCE | | |

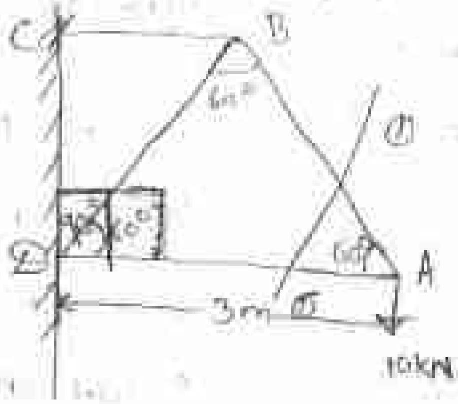
30 Sep 2020

3Q find the axial forces in the members of given truss is shown in figure.




(By using method of section)

Soln

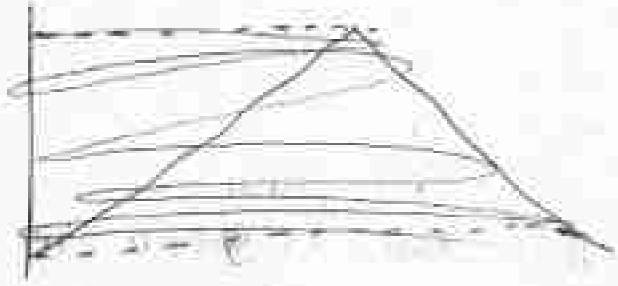


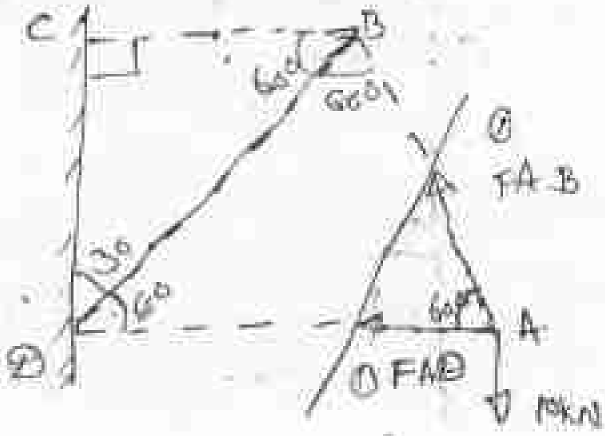

$m\angle CDA = 90^\circ$

$m\angle CDB = 90^\circ - m\angle BDA = 90^\circ - 60^\circ$
$= 30^\circ$

Consider section ① - ①

Let us consider the right part of the truss of the section ① - ①






$Ax = 3\cos 60 = 1.5m.$

$Bx = 3\sin 60^\circ$
$= 2.598m.$

To find FAD $\Sigma M_B = 0$

$\Rightarrow 10kN \times 3\cos 60^\circ + FAD \times 3\sin 60^\circ = 0$

$\Rightarrow 10 \times 1.5 + F_{AD} \times 2.598 = 0$

$\Rightarrow F_{AD} = \dfrac{-10 \times 1.5}{2.598} = -5.7 \text{ KN}$

$F_{AD} = 5.75 \text{ KN}$ (compressive)

To find $F_{AB}$ $\Sigma M_D = 0$

$\Rightarrow -F_{AB} \times DZ + 10 \times 3 = 0$

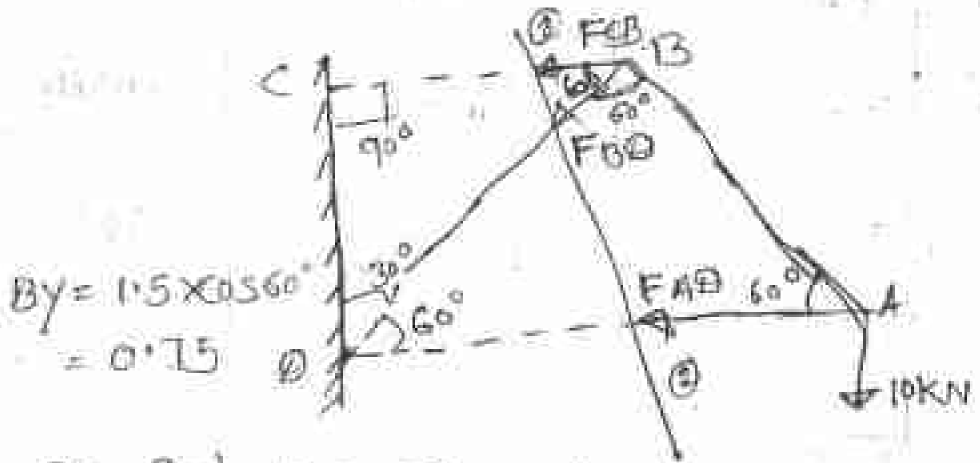
$\Rightarrow F_{AB} \times 2.598 = 30 \text{ KN}$

$F_{AB} = \dfrac{30}{2.598} = 11.54 \text{ kN}$.

$F_{AB} = 11.54 \text{ KN}$ (Tensile)

Consider Section ②-②

Let us consider the right part of the truss of the section ②-②




To find $F_{CB}$, $\Sigma M_D = 0$

$\Rightarrow -F_{CB} \times CD + 10 \times 3 = 0$ $\quad CD = BX$

$\Rightarrow F_{CB} \times 3 \sin 60° = 10 \times 3$

$\Rightarrow F_{CB} \times 2.598 = 30$

$\Rightarrow F_{CB} = \dfrac{30}{2.598} = +11.54 \text{ KN}$ (Tensile)

To find FBD, $\Sigma M_C = 0$

$\Rightarrow -\left[(10\times3) + (FBD \times CF) + (FAD \times CD)\right] = 0$

$\Rightarrow (10\times3) + (FBD \times 1.29) + (-5.75 \times (2.592)) = 0$

$$FBD = \frac{(5.75\times2.592) - (10\times3)}{1.29}$$

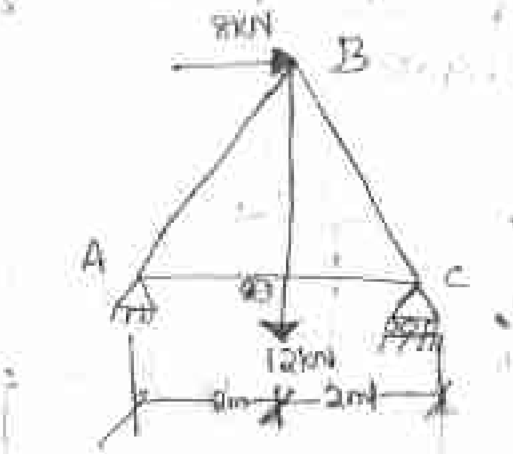
=

$\Rightarrow FBD = -11.67$ kN $= 11.67$ kN (compressive)

| Sl.no | ~~Nature~~ of forces in the members | Magnitude | Nature |
|---|---|---|---|
| (i) | FAB | 11.54 kN | Tensile |
| (ii) | FAD | 5.75 kN | Compressive |
| (iii) | FBC | 11.54 kN | Tensile |
| (iv) | FBD | 11.67 kN | Compressive |

Q3 1 ... 2023

A frame of 4m span and 1.5m high subjected to two point loads at B and D as shown in the figure. Find the forces in all the members of the structure by method of sections and method of joints and compare both the method




Sol^n

Step - I

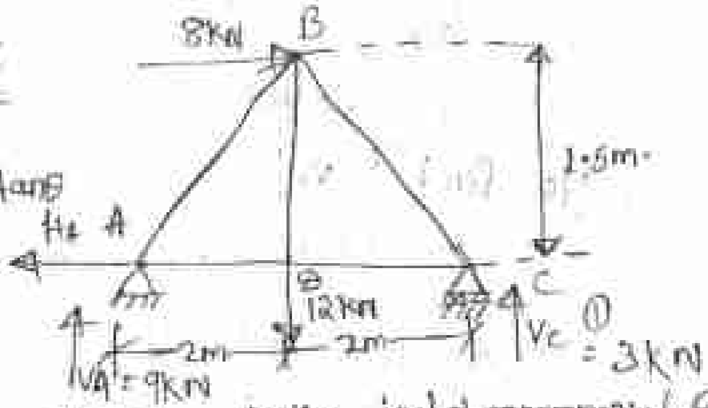
$\tan\theta = 0.75$

$\theta = \tan^{-1}(0.75)$ tanθ

$\tan\theta = \frac{BD}{AD}$

$= \frac{1.5}{2}$

$\theta = 36.9^{\circ}$




Total static indeterminacy ($D_s$)

$= D_{se} + D_{si}$

$D_{se}$ = external static indeterminacy

$= r_e - 3$

$r_e \rightarrow$ external reaction

Total external reaction $= 2 + 1 = 3$

$D_{se} = 3 - 3 = 0$

(So the truss is externally)

$D_{si}$ = internally static indeterminacy

$D_{si} = m - 2j + 3$

$m \longrightarrow$ no of members.

J → No of joints.

$Dsi = m - 2J + 3$

$= 5 - 2\times4 + 3$

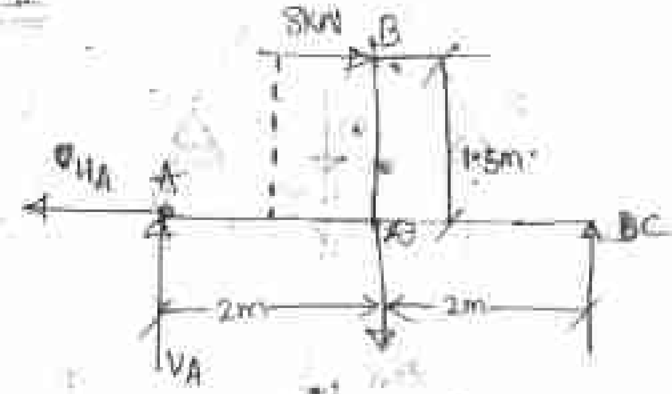
$= 5 - 8 + 3 = 0$

So the truss is internally determinate

$Ds = Dse + Dsi = 0 + 0 = 0$

So the truss is statically determinate structure.

Step-II



So $H_A = 8KN (\leftarrow)$

To find '$V_c$'

Taking moment at 'A', $\Sigma M_A = 0$

$\Rightarrow$ T.A.C.M = T.C.C.M

$\Rightarrow V_c \times 4 = 8 \times 1.5$

$\Rightarrow V_c = \frac{8 \times 1.5}{4} = 3KN (\uparrow)$

To find $V_A$

Total upward load = Total downward load

$\Rightarrow V_A + V_c = 12KN$

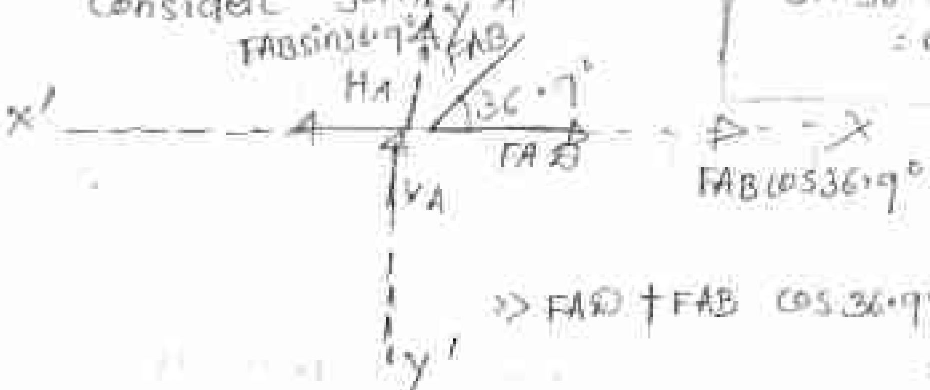
$\Rightarrow V_A = 12 KN - 3 KN = 9 KN (\uparrow)$

Step - III (Method of joint)

$\cos 36.9° = 0.8$

$\sin 36.9° = 0.6$

Consider joint 'A'




$\Rightarrow F_{AD} + F_{AB} \cos 36.9° - H_A = 0$

$\Rightarrow F_{AD} + F_{AB} \times 0.8 - 8 = 0$ ——— ①

$\Sigma F_y = 0$

$\Rightarrow F_{AB} \sin 36.9° + V_A = 0$

$\Rightarrow F_{AB} \sin 36.9° = -9 \text{ kN}$

$\Rightarrow F_{AB} = -\frac{9}{0.6} = -15 \text{ kN}$

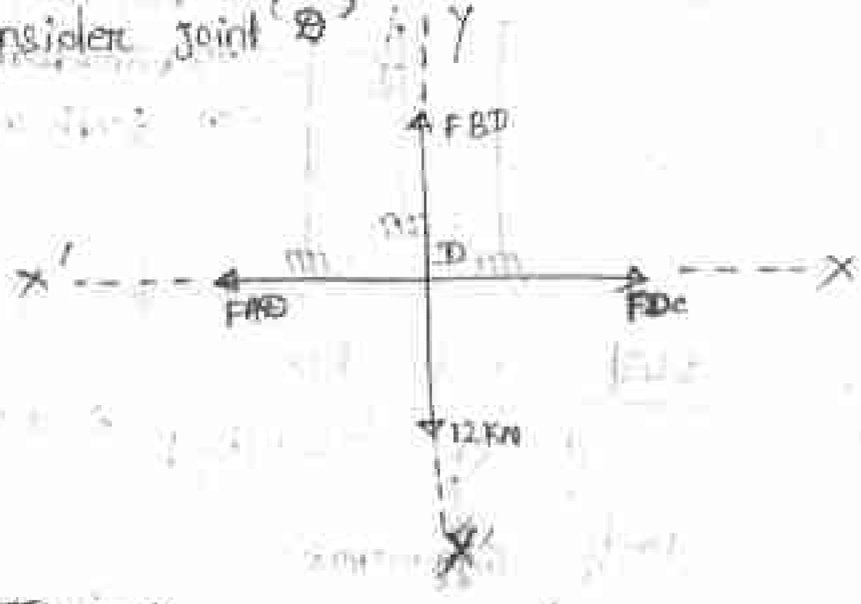
$= 5 \text{ kN}$ (Compressive)

put the value of $F_{AB}$ in eqn ①

$\Rightarrow F_{AD} + (-5 \text{ kN}) \times 0.8 - H_A = 0$

$\Rightarrow F_{AD} = 8 + (5 \times 0.8) - 12 \text{ kN}$ (Tensile)

Consider joint 'D'




$\Sigma F_x = 0$

$F_{DC} - F_{AD} = 0$

$F_{DC} = F_{AD}$

$F_{BC} = 12\ KN$ (Tensile)

$\Sigma F_y = 0$

$F_{BD} - 12 = 0$

$\Rightarrow F_{BD} = 12 KN$ (Tensile)

3 Oct 2020

## Structural Mechanics of solid

**Mechanics :-** The study of forces, energies and their effects on various type of body is known as mechanics.

**Structure :-** It is a body composed of various structural elements such as beam, column, slab, footing etc. which can set up resistance against deformation with the application of external force.

(i) Dead load (self wt)
(ii) Live load
(iii) Wind load
(iv) Earth quake load

**Solid** is two types

(i) Rigid solid / body (ii) Deformed solid / body

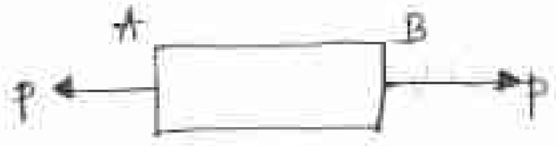
Rigid solid / body → Engg mechanics; Load → No deformation takes place

Deformed solid / body → SM / M.O.M / MODM; Load → Deformation takes place

(MODB → Mechanics of Deformed body)

## Structural mechanics of solid :-

The study of forces, energies and their effect on various type of deformed body and under goes change in physical mechanical propoties and appearance (dimension).




Force

**Mechanical force :-** The force ~~applied~~ which is applied by direct physical contact is called as mechanical force.

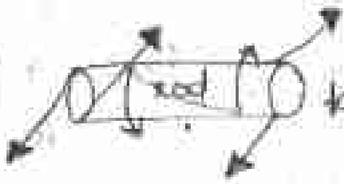
### Effect of force :-

(i) 


(Tensile force)
pull → extension

(ii) 

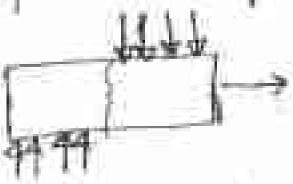

(Compression force)
push → compression

(iii) 

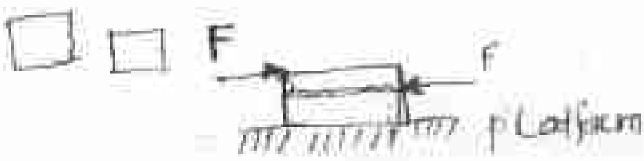

couple of force → twist
Torque = (F × ⊥)

(iv) 


Bend

(v) 

→ Brake or Shear




Condition of equilibrium

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma M_z = 0$$

To safity the condition of equilibrium the ressisting force will be equal manghtde but opposite direction.

$$\Sigma F_x = 0$$

$$\Rightarrow P - f = 0$$

$$\Rightarrow \boxed{P = f} \text{ (Resisting force)}$$

**Stress:-**

* stress is defined as resisting force per unit area.
* It is denoted by $\sigma$ (sigma)
* Mathmaticaly stress = $\dfrac{\text{Resisting force}}{\text{Area}}$

$$= \boxed{\sigma = \frac{f}{A}}$$

* The S.I unit of stress is $N/m^2$.

$$1\ N/m^2 = 1 \text{ pascal}$$

$$1\ pa = 1\ N/m^2$$

Kilo = $10^3$ k

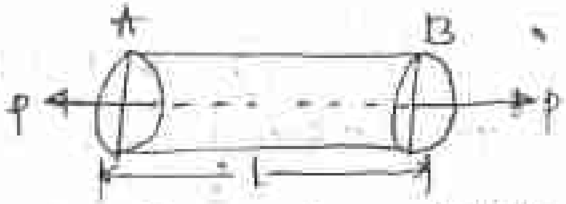
Mega = $10^6$ M

Giga - $10^9$ G

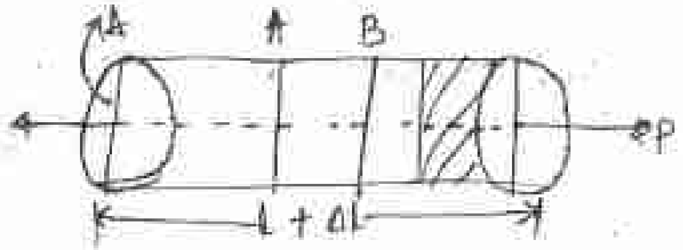
Terra - $10^{12}$ T
Milli - $10^{-3}$ M
Micro - $10^{-6}$ M
Nano - $10^{-9}$ N
Pica - $10^{-10}$ P

Simple Stress and Strain.

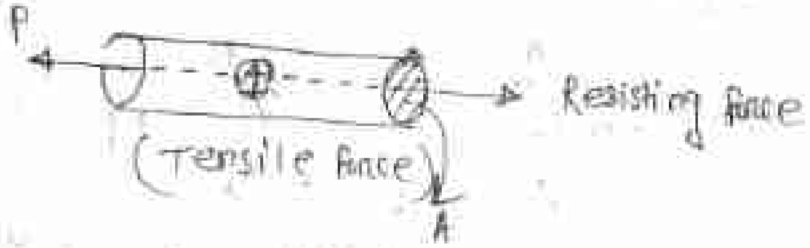
Let us consider a bar of length 'l' and having c/c area of 'A' and a tensile load of 'P' is acting its longitudinal axis.

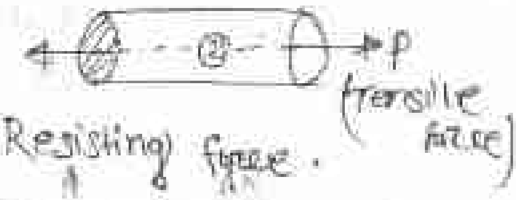



Due to the action of this external force (tensile). The length of bare its increased to 'A l'




Consider two fibres 'A' and 'B'




Tensile stress $(\sigma_t)$ = $\frac{\text{Tensile force}}{\text{Area}}$

$= \frac{P}{A}$

Compressive stress: The stress induced in a body when subjected to two equal and opposite pushing. As a result of which there is a decrease in length of the body. is known as Compressive stress.


Compressive stress $(\sigma_c)$

$= \dfrac{\text{Compressive load}}{\text{Area}}$

$$= \sigma_c = \frac{P}{A}$$

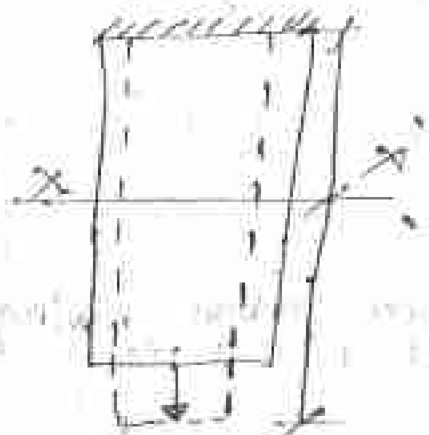
Shear stress :- The stress induced in a body when subjected to two equal and opposite forces which are acting tangentially across the resisting section. As a result of which the body tend to shear off across the section is known as shear stress.

6-1-25

Strain When a body is subjected to some external force there is change in dimension of the body.

Def The ratio of change in dimension of the body to the original dimension is known as strain.

* Mathematically $\varepsilon = \dfrac{\text{change in dimension}}{\text{original dimension}}$

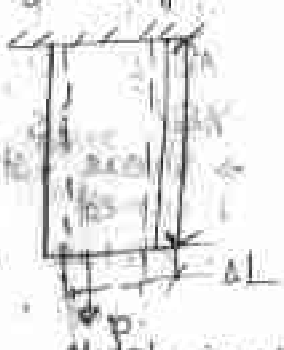


$\varepsilon = \dfrac{\delta l}{l}$

There are four type of strain:-

(i) Longitudinal strain
(ii) Lateral strain
(iii) Volumetric strain
(iv) Shear strain

(1) Longitudinal Strain - It is the ratio betn change in length to it's original length



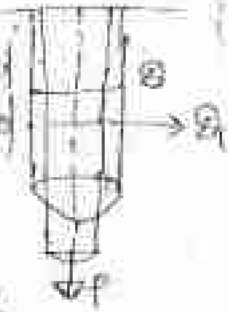
So ~~lengte~~ longitudinal strain $= \dfrac{\delta l}{L}$

$= \dfrac{\text{change in lengths}}{\text{original length}}$

The unit of longitudinal strain is unitless quantity.

Lateral strain Application of deforming force is one direction results in change in parameters in direction perpendicular to it.

Consider a cylinder having its one end fixed to a rigid support let force 'P' is applied at it's free end in downward direction.



=> The length of cylinder small increase while it's diameter will decrease.

=> Lateral strain- The ration between change to it's original diameter when the cylinder is subjected to a force along it's longitudinal axis.

=> Volumetric strain :-

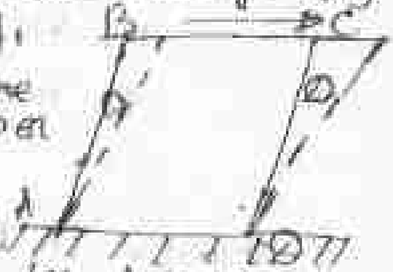
Defn It is defined as the ratio betn change in volume, it's original volume

* mathematically = $\frac{V-x}{V}$ = volumetric strain

Consider a ball having a volume 'V' compressed to a position shown in the & the decrease volume is x, volumetric strain

$= \frac{V-x}{V}$

③ Shear Strain :- Let ABCD be the cross sectional view of cube having it's Lower face 'AD' is fixed.



=> Apply a force ~~strange~~ tangentially to the upper face. The cube get deshaped to the position ABCD Angle ϕ turned by the line 'AD' is the measure of shear strain

=> shear strain is measured by angle turned by a line originally perpendicular to the fixed face

$\tan\phi = \frac{DD'}{AD}$ $\phi$ << small $\tan\phi = \phi$

Shear Strain ($\phi$) $= \frac{DD'}{}$

$= \frac{\text{Displacement Inplane CD}}{\text{Displacement of Plane CD from fixed place.}}$

Shear strain is also defined as the ratio betn displacement in one plane to its distance from the fixed plane.

Mechanical properties of deformable solid

Mechanical properties – The property which can be determined or observed by the application of mechanical force or energy.

Mechanical force – A force which requiring direct physical contact that is called as mechanical force.

These are the following mechanical properties as follows

① Elasticity – It is the property of material by virtue of which it returns back its original position after removal of external force. The body is known as Elasticity body.

7 Oct 2020

(2) Strength :- The maxm value of stress which can sustain without failure.

Failure

↓ Due to plastic deformation

↓ Permanent rupture / fracture (Brittle metal) rod → ductile material

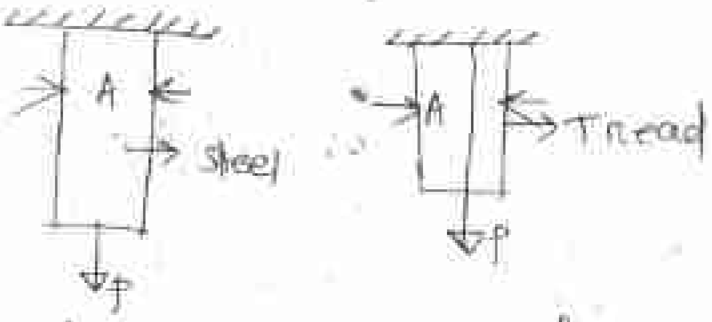
Chalk → Brittle material

5-15% elongation → Intermediate ductile.

>15% elongation → Completely ductile material

<5% elongation → Brittle material

Stress doesn't depend upon the material.



$\sigma_S = \frac{P}{A}$ $\sigma_T = \frac{P}{A}$

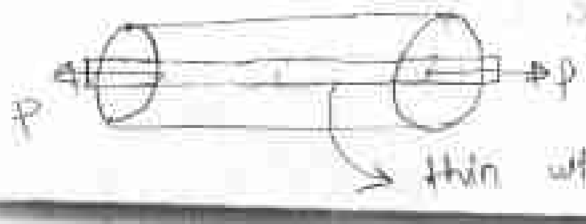
Factor of safety :- (Fos)

$$F.O.S = \frac{\text{Strength}}{\text{Design stress}} = \frac{\text{Failure stress } (N/m^2)}{\text{permissible stress } (N/m^2)}$$

* Dimension or unitless quantity

(3) Ductility and brittleness :-

Ductility :- It is the property of material by virtue of which can drawn into thin wire.



Brittleness :- It is the property of material by virtue of which a material will undergo low degree of deformation before fracture.

Due to brittleness :

→ Low reduction in c/s area.

→ It is very difficult to draw the material into thin wire.

(4) Malleability :- It is the property of material by virtue of which can drawn into thin sheet.

Due to malleability

(i) Large reduction in c/s area.

(ii) High degree of plastic deformation.

(5) Tougness :- It is the property of material by virtue of which a material absorbs maximum amount of energy before fracture.


$\boxed{T_W > T_G}$

Resilience and proof resilience

xox

Strain energy (U) :- The energy stored in a body due to deformation.

Strain energy (U) = Work done due to displacement

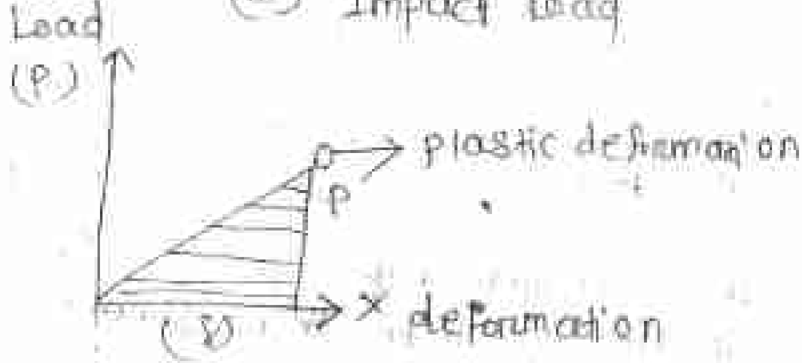
Load for Strain energy (U)

↓

(i) Gradual load

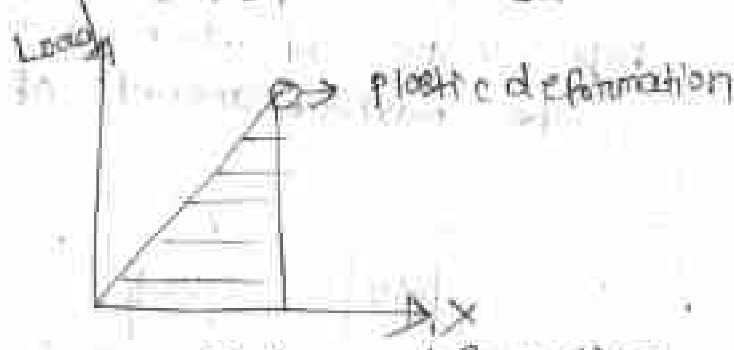
(ii) Sudden load

(iii) Impact load



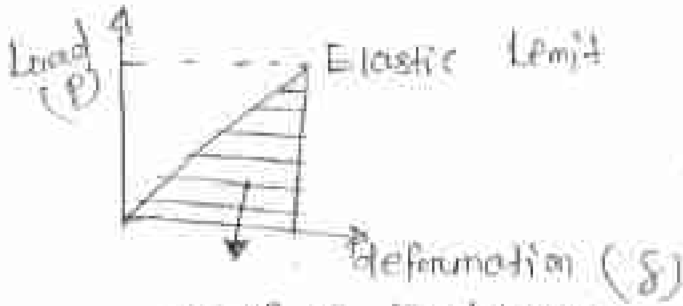

Strain energy $= \frac{1}{2} \times \delta \times P$

(6) <u>Resilience</u>:- The energy observed by a component within elastic limit is called as resilience.




Resilience

<u>Proof resilience</u>:- Area under load Vs deformation diagram up to elastic limit is called as proof resilience.




proof of Resilience

(7) Stiffness :- It is the property of material by virtue of which a material resist deformation is known as stiffness.

| A | B |
|---|---|
| 1mm ← 100KN | 100KN = 1m.m |
| 2mm ← 200KN | 200KN = 1m.m |
| | $(S_B > S_A)$ |

(8) Plasticity :- It is the property of material by virtue of which a material tends to deform permanently.

(9) Hardness It is the resistance against penetration.

(10) Creep :- It is the property due to which a material deform continiously action of a dead load (constant stress) at an elevated a constant temp.

Hooke's Law :-

It states that within elastic limit the stress is directly proportional to strain.

Mathmetically Stress $\propto$ strain

$\Rightarrow$ Stress $= a$ Strain

where $a \Rightarrow$ proportionality constant is called as modulus of elasticity.

$$\Rightarrow a = \frac{\text{Stress}}{\text{Strain}}$$

According to modulus of elasticity is divided into 3 parts

(a) Young's modulus (E)

(b) Modulus of rigidity $(C, G, N)$

(a) Young's modulus (E) :- It is the ratio between tensile stress to the tensile strain. or compressive stress to compressive strain.

* It is denoted by E

$$E = \frac{\sigma}{\epsilon}$$

(b) Modulus of regidity :- It is the ratio betn shear stress ($\tau$) to the shear strain.

* It is denoted by (C, N, G)

$$C, N, G = \frac{\tau}{e_s} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

Bulk modulus of Elasticity :- (K)

It is defined as the ratio of normal stress ($\sigma_N$) to the volumetric strain.

* It is denoted by 'K'

$$K = \frac{\sigma_N}{e_V} = \frac{\text{Normal stress}}{\text{Volumetric strain}}$$

Yong's modulus :- (E)

It is the ratio between axial / longitudinal stress to axial / Longitudinal strain.

$$E = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

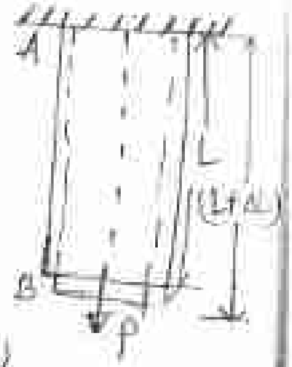
The value of 'E' fore different material

(1) - Steel ⟶ E = 200 to 220 GPa
= (200 to 220) $\times 10^3$ N/mm²
= (200 to 220) GN/m²

(2) Wrought iron - E = (190 to 200) GPa
(3) Cast iron - E = (100 to 160) GPa
(4) Copper - E = (90 - 110) GPa
(5) Brass - E = (80 - 90) GPa
(6) Wood - E = 10 GPa

Deformation of a body due to force acting on

Consider a body subjected to tensile Load.




Let $P \rightarrow$ load acting on it

$L \Rightarrow$ length of the body

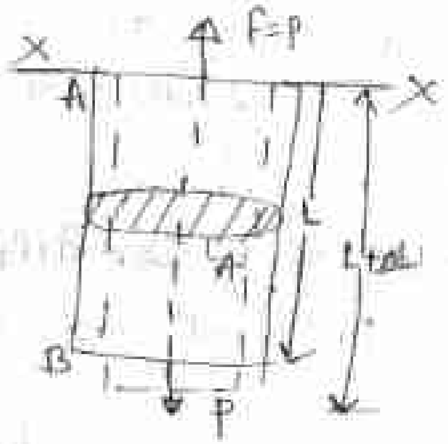
$A \Rightarrow$ Cross sectional area of the body

$\sigma \rightarrow$ Stress induced in the body

$E \rightarrow$ Modulus of elasticity for the material of the body.

$\varepsilon \Rightarrow$ strain in the body

$\Delta L \rightarrow$ Deformation of the body.




'E' $L+\Delta L - \Delta L$

Stress $\sigma = \dfrac{\text{Resisting force}}{\text{C/S area}}$ $(P = F)$

$$\sigma = \frac{F}{A} = \frac{P}{A}$$

$$\sigma = \frac{P}{A} \quad \text{---} \quad (1)$$

Young's Modulus $E = \dfrac{\text{longitudinal stress}}{\text{longitudinal strain}}$

Longitudinal strain $= \dfrac{\text{Change in length}}{\text{original length}}$

longitudinal strain $= \frac{\Delta L}{L}$

$$\Rightarrow \varepsilon_L = \frac{\Delta L}{L} \quad \text{--- (II)}$$

from eqn (I) and (II)

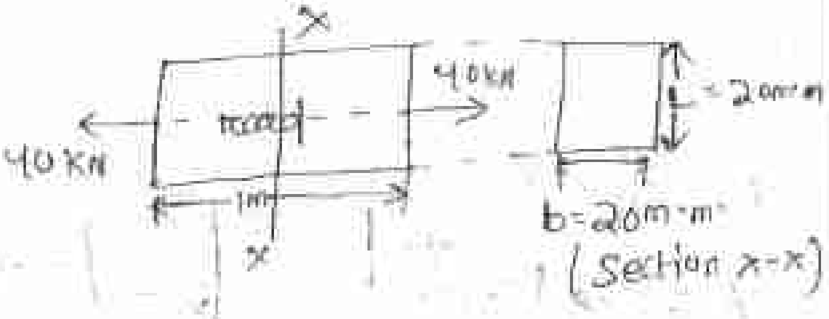
$$E = \frac{\frac{P}{A}}{\frac{\Delta L}{L}} \Rightarrow \frac{\Delta L}{L} = \frac{P/A}{E}$$

$$\Rightarrow \frac{\Delta L}{L} = \frac{PL}{AE}$$

$$\boxed{\Delta L = \frac{PL}{AE}}$$

Q: A steel 1mt long and 20mm×20mm in cross section is subjected to a tensile force of 40 kN. Determine the elongation of rod if E = 200 GPa

Soln :-




Given data:- Width of the rod (b) = 20 mm.

Depth of the rod (d) = 20 mm.

Length = 1mt = 1000 mm.

tensile pull (P) = 40 kN

Young's modulus of steel

$E = 200$ GPa

$= 200\times10^9$ pa

$= 200\times10^9$ $[1\text{pa} = 1\text{N/m}^2]$

$= \frac{200\times10^9}{10^6}$ N/mm$^2$

$1\text{pa} = 1\text{N/m}^2$

$1\text{m} = 1000\text{mm}$

$1\text{m}^2 = 1000\times1000\text{mm}^2$

$= 10^6\text{mm}^2$

$$E = \frac{200\times10^9}{10^6} \text{ N/mm}^2$$

$= 200\times10^3$ N/mm$^2$ $\qquad \frac{a^x}{a^y} = a^{x-y}$

$$\Delta L = \frac{PL}{AE}$$

$$A = b \times d = 20 \times 20 = 400 \text{ mm}^2$$

$$\Rightarrow \Delta L = \frac{400 \times 10^3 \times 1000}{400 \times 200 \times 10^{-3}}$$

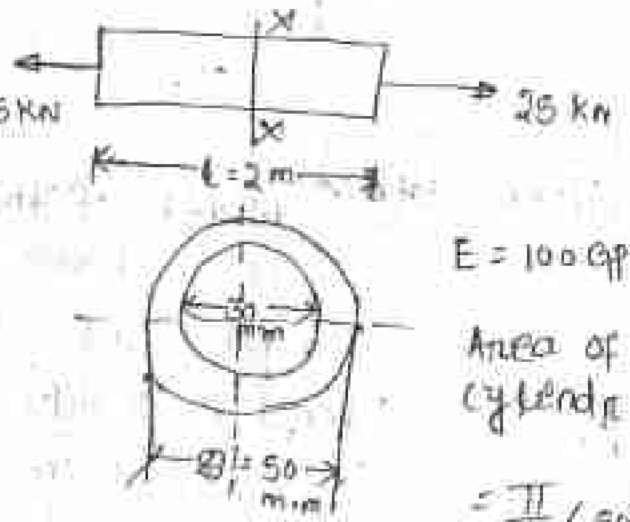
$$= 0.5 \text{ m.m.}$$

[illegible] A unique entry

Q. A hollow cylinder 2m long has an outer diameter 50mm and inside diameter 30mm. If the cylinder is carrying a load of 25 kN find the stress in the cylinder. If the value of modulus of elasticity of the cylinder material is 100 GPa & find elongation in the cylinder.

$A_1 = \frac{\pi}{4} \times D_1^2$

$A_2 = \frac{\pi}{4} \times D_2^2$ $P = 25$ kN




E = 100 GPa.

Area of hollow cylinder $= \frac{\pi}{4}(D_1^2 - D_2^2)$

$= \frac{\pi}{4}(50^2 - 30^2)$

$= 1257 \text{ mm}^2$

Soln

Data given:-

Length of the cylinder (l) = 2m. = 2000 m.m.

Load = 25 kN = $25 \times 10^3$ N

outer dia of cylinder ($D_1$) = 50 m.m.

Inner dia of cylinder ($D_2$) = 30 m.m.

Modulus of elasticity $(E) = 100\,GPa = 100 \times 10^3\ N/mm^2$

$1\,Pa = 1\,N/m^2$

$100\,GPa = 100 \times 10^9\,N/m^2$

$1\,m = 1000\,mm$

$1\,m^2 = 10^6\,mm^2$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$100\,GPa = \frac{100 \times 10^9}{10^6}\,N/mm^2$$

$$100\,GPa = 100 \times 10^{9-6}\,N/mm^2$$

$$= 100 \times 10^3\,N/mm^2$$

$$\text{Stress}(\sigma) = \frac{\text{Resisting force}}{\text{Area}}$$

Resisting force = applied load $(F = P)$

$$= \frac{F}{A}$$

$$= \frac{P}{A}$$

$$= \frac{25 \times 10^3}{1257} = 19.88\,N/mm^2$$

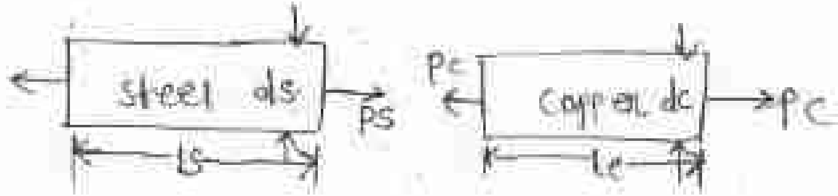
$$\Delta L = \frac{PL}{AE}$$

$$= \frac{25 \times \cancel{10^3} \times 2000}{1257 \times 100 \times \cancel{10^3}} = 0.397\,mm$$

The elongation in hollow cylinder is 0.397 mm.

**Q3** Two wires, one of steel and other copper are of same length and are subjected to same tension. If the diameter of copper wire is 2mm, find the dia of steel wire is 2mm, find the dia of steel wire, if they are elongated by the same amount, take $E_{steel} = 200\,GPa$.

$E_{copper} = 100$ Gpa




$(L) = L_s = L_c$, $P_s = P_c = (P)$, $d_c = 2mm$

$\Delta L_s = \Delta L_c = (\Delta L)$ $d_s = ?$

$\Delta L_s = \frac{P_s L_s}{P_s E_s}$ $E_s = 200$ Gpa

$= \frac{P_s L_s}{A_s 200\,Gm}$ $= 200 \times 10^3\, N/mm^3$

$E_c = 100$ Gpa

$100 \times 10^3 N/mm^2$

$= \frac{P_s L_s}{(\frac{\pi}{4} d_s)^2} \times 200 \times 10^3$

$= \frac{P_s L_s}{(\frac{\pi}{4} \times d_s^2) \times 2}$

$$\left[\begin{aligned} A_c &= \frac{\pi}{4} \times d_c^2 \\ A_c &= \frac{\pi}{4} \times 2^2 \\ &= 90.8 \end{aligned}\right]$$

Elongation in

$\Delta L_c = \frac{P_c L_c}{A_c E_c}$

$= \frac{P_c L_c}{\frac{\pi}{4} \times 2^2 \times 100 \times 10^3}$

$= \Delta L_s = \Delta L_c = (\Delta L)$

$\therefore \Delta L = \frac{P_s L_s}{\frac{\pi}{4}(d_s^2) \times 200 \times 10^3}$

$\Delta L = \frac{P_c L_c}{\frac{\pi}{4} \times 2^2 \times 100 \times 10^3}$

$$\Rightarrow \frac{P_s l_s}{\frac{\pi}{4}(d_s)^2 \times 200\times10^3} = \frac{P_c l_c}{\frac{\pi}{4}\times 2^2\times 100\times10^3}$$

$$\Rightarrow \frac{pl}{d_s^2\times 200} = \frac{pl}{2^2\times 100}$$

$$\begin{bmatrix} P_s = P_c = P \\ l_s = l_c = l \end{bmatrix}$$

$$= d^2 \times 200 = 4\times 100$$

$$\Rightarrow d_s = \frac{4\times100}{200}$$

$$= \frac{400}{200} = 2$$

$$\Rightarrow d_s = \sqrt{2}$$

12 oct 2020

Consider a bar AB having freely under its own weight as shown in th fig.

Let L $\longrightarrow$ Length of the bar.

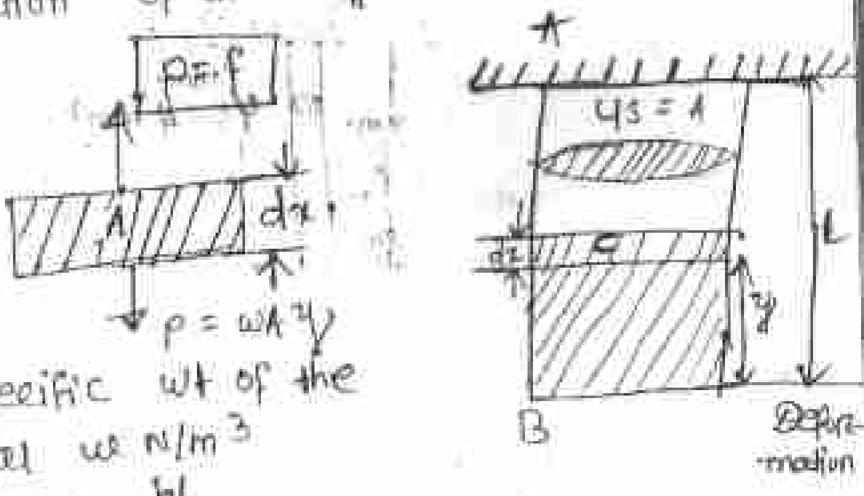
A $\longrightarrow$ Area of c/s of the bar

E $\longrightarrow$ youngs modulus of elasticity

w $\longrightarrow$ sp. wt / unit wt of the material

Now consider a small strip dx along the length of the bar at a distance 'y' from free end.

Deformation of a body due to self wt:-




Specific wt of the material w N/m³

$$w = \frac{W}{V}$$

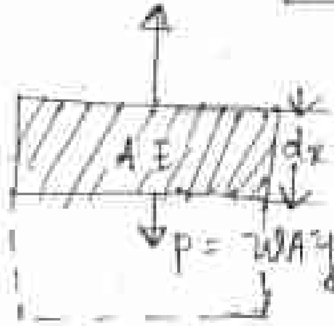
$$\Rightarrow W = w\times V = w\times A\times c = w\times A\times y$$

Wt of the bar for length of 'y'

unit wt / sp. wt (w) $= \frac{W}{V}$

$\Rightarrow W = wV$

$f = P \Rightarrow \boxed{W = w \times A \times y}$

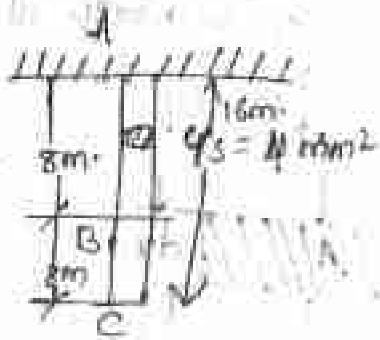



Elongation of the small strip due to the wt of the bar for length of 'y'

$\Delta L = \frac{PL}{AE}$

$= wAy\,dy$

Q1 A steel wire 16 m long having c/s area of 4 mm² weights 20 N as shown in figure. If the modulus of elasticity (E) for the wire material is 200 GPa find deflection at 'C' and 'B'.




$\Sigma F_x = 0$

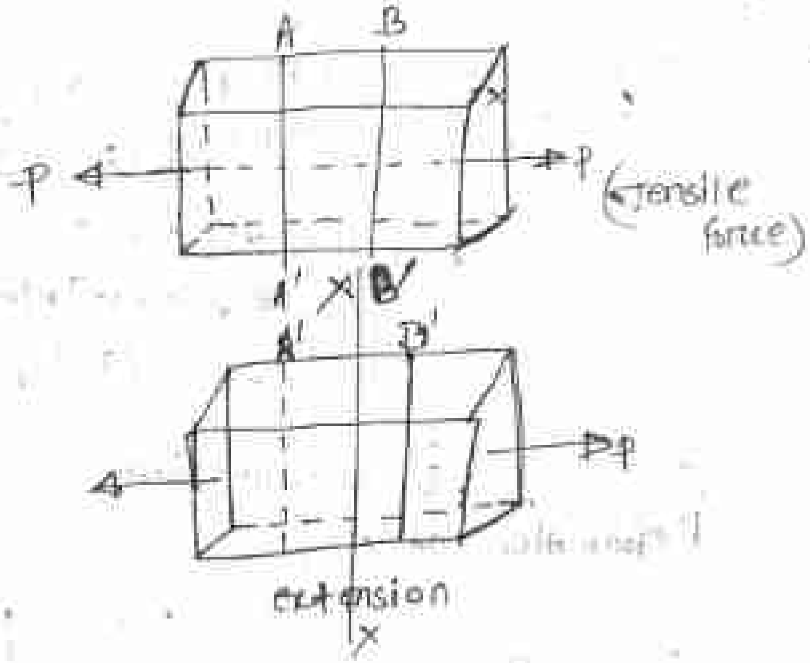
$\Rightarrow P - F = 0$

$\Rightarrow \boxed{P = F}$ (Resisting force)

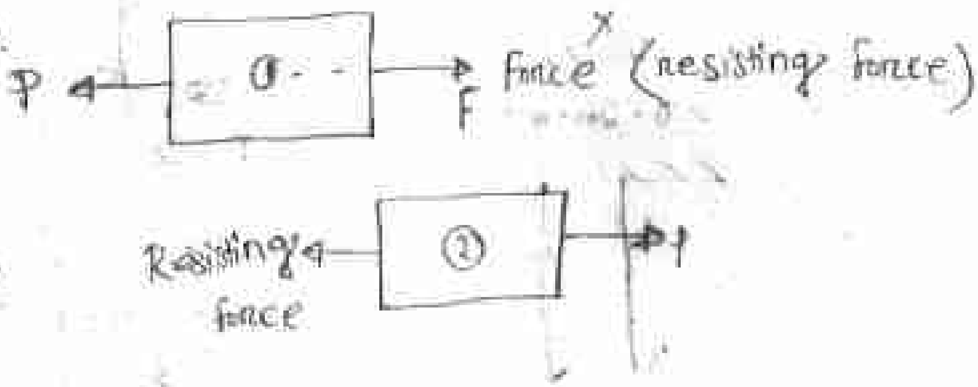
## Stress :-

* Stress is defined as resisting force per unit area.
* It is denoted by $\sigma$ (sigma)
* Mathmatically stress = $\dfrac{\text{Resisting force}}{\text{Area}}$

## Simple stress and strain :-




## Method of section :-



To safity the condition of equilibrium. The resisting force will be equal magnitude but opposite direction

Condion of equilibrium

$\Sigma f_x = 0$

$\Sigma f_y = 0$

$\Sigma M_z = 0$

Soln

Step 1

Given data

Total length of the wire = 16m

C/C area 'A' = 4mm², wt of the wire(w) = 20N = 1600mm.m

modulus of elasticity (E) = 200 Gpa

$= 200\times10^9$ pa

$= 200\times10^9\ N/m^2$

$= 200\times10^{(9-6)}\ N/mm^2$

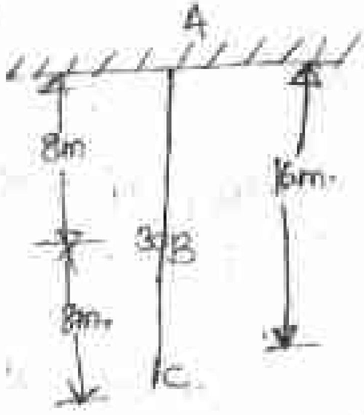
$E = 200\times10^3\ N/mm^2$

Deformation at 'c'

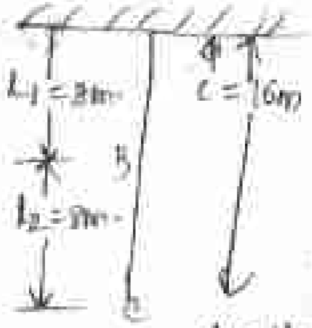
Elongation due to self wt $\Delta L = \frac{WL}{2AE}$

$= \frac{20\times16\times10^3}{2\times4\times200\times10^3}$

$= 0.2mm$




$E = 200\times10^3\ N/mm^2$


A = 4mm²

$E = 200\times10^3\ N/mm^2$

$w = \frac{20}{2} = 10N$

$L = L_1 + L_2$, A = 4 mm²

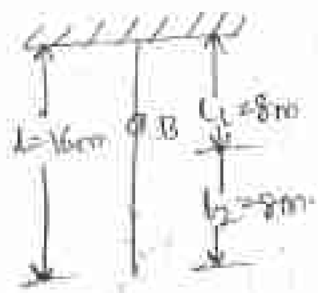
w = 20N

Deformation of 'AB' (due to self wt.)

$$\Delta L = \frac{wL_1}{2AE}$$

$$= \frac{10\times 8\times 10^3}{2\times 4\times 200\times 10^3}$$

$$= 0.05\ m.m$$






for long the of

$$8m = \frac{20}{2} = 10$$

$$\Delta L = \frac{PL}{AE}\ \text{(due to wt of BC)}$$

$$= \frac{10\times 8000}{4\times 200\times 10^3} = 0.1\ m.m$$

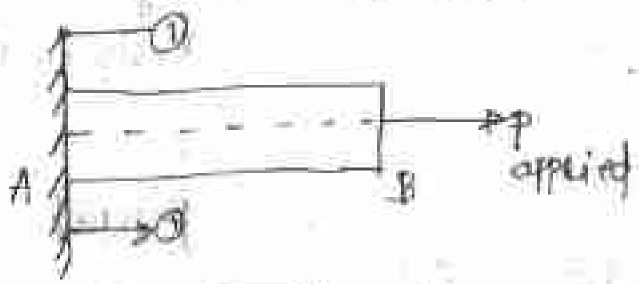
Total deflection at 'B' $= \Delta L + \Delta L$

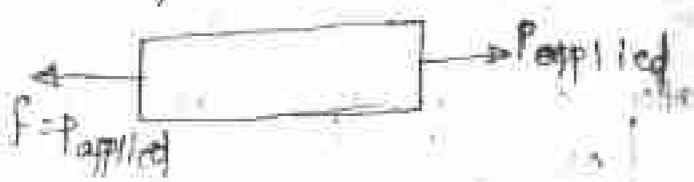
(for self wt) (for the wt of BC)

Oct 3 2020 $= 0.05+0.10 = 0.15\ m.m$

Ans

Members in series and parallel:-



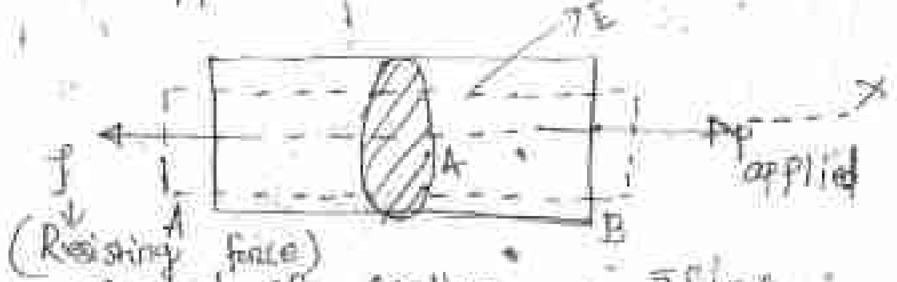




Let us consider body 'AB' which is fixed at one end 'A' and free at other end as shown in the figure.

Let $P$ be the applied force acting on it (Fixed end). $L \rightarrow$ length of body / bar / member.

A little consideration we'll see that if we increase the the applied force (P) then the resisting force will increase at 'A'.

The free body diagram of the body at 'A'




Method of section

$\Sigma f_x = 0$
$\Sigma f_y = 0$
$\Sigma M_z = 0$

$\Sigma f_x = 0$

$\Rightarrow P_{applied} - f = 0$

$\Rightarrow \boxed{f = P_{applied}}$

Let the C/S of the body = A

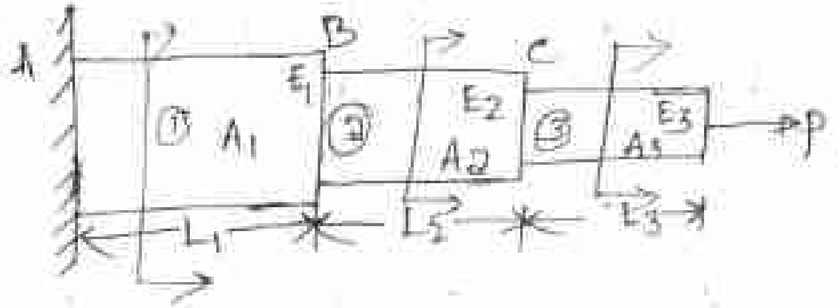
Modulus of elasticity = E

$$\text{Stress } (\sigma) = \frac{\text{Resisting force}}{\text{Area}} = \frac{P_{applied}}{A}$$

Elongation due to axial load

$$\boxed{\Delta L = \frac{PL}{AE}}$$

If no. of bars are so connected end to end of different length & different modulus of elasticity.




Members in series :-

① End to end connection

② Load is same in all the members if a single load is applied at its extreme end

③ Total change in length will be

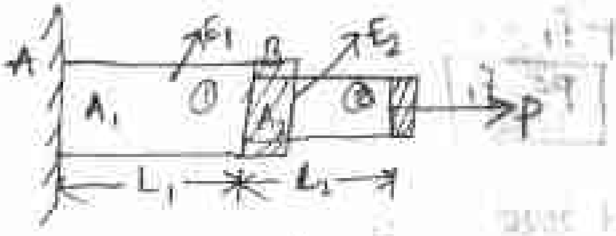
$$\Delta L_T = \Delta L_1 + \Delta L_2 + \Delta L_3 \cdots \Delta L_N$$

$$= \Sigma (\Delta L (1 \cdots n))$$

(This formula is called as principle of superposition.)

Principle of superposition -

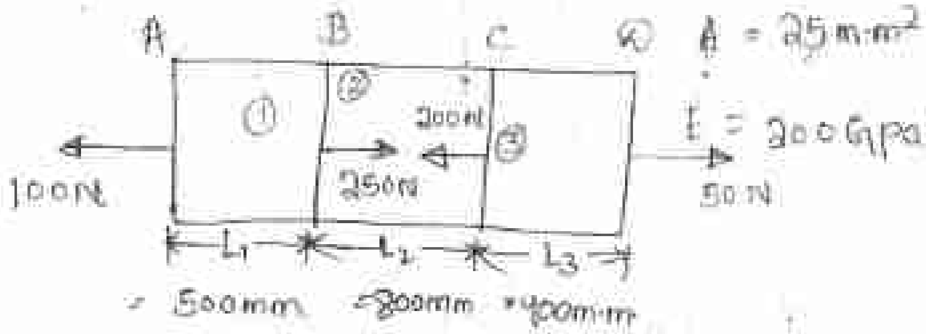
The resulting deformation of composite body is equal to the algebric sum of the deformation of the individual section.



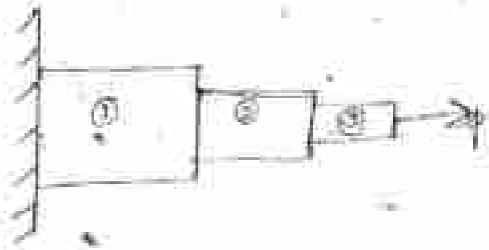

$$\Delta L_T = \Delta L_1 + \Delta L_2$$

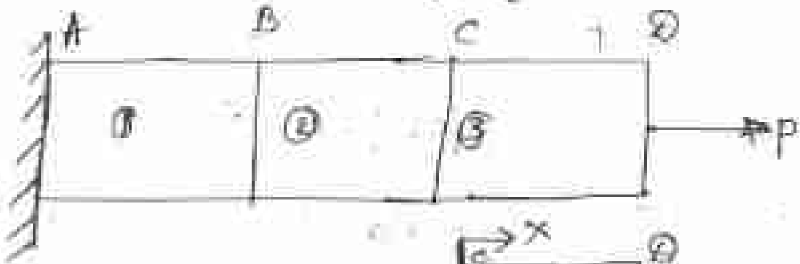
$$= P\left[\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2}\right]$$

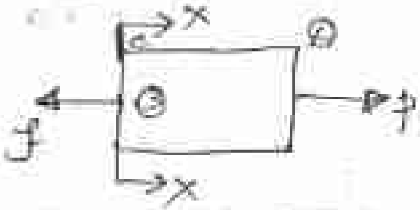
**Q1** Find the change in length of a prismatic bar as shown in the figure.

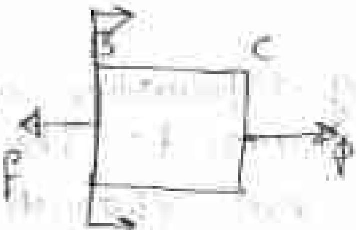



$\Delta L_T = ?$




$\Sigma F_x = 0$

$\Rightarrow P - f = 0$

$\Rightarrow \boxed{P = f}$

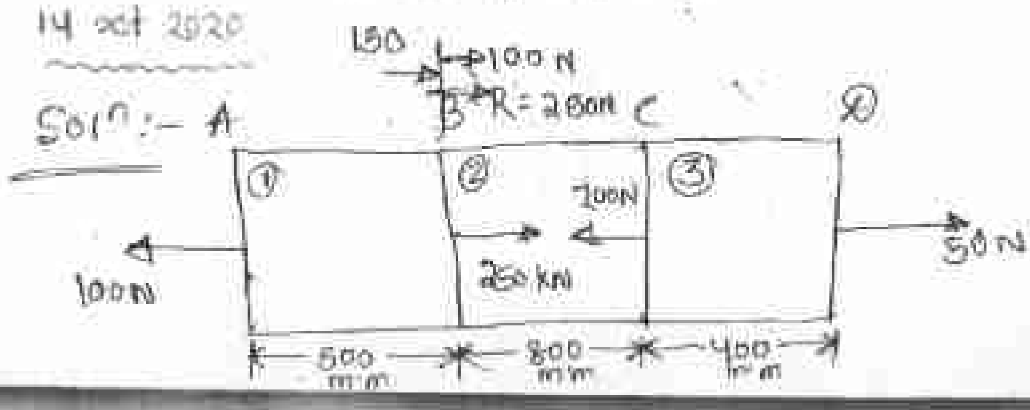
$\Sigma F_x = 0$

$P - f_1 = 0$

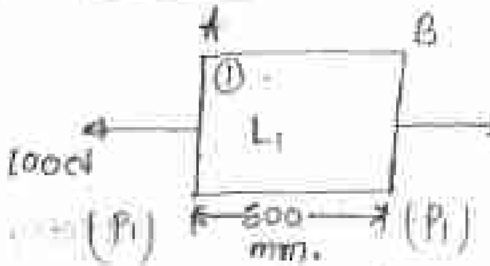
$\boxed{P = f_1}$

14 oct 2020

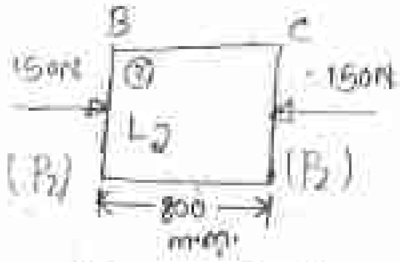
**Soln:-**



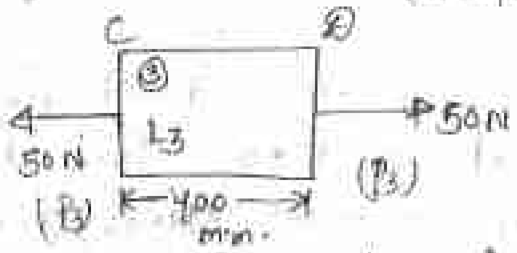
F-B-D




(Extension)




(Compressive)




(Extension)

Given data :-

$A = 25\ mm^2$

$E = 200\ GPa$

$= 200 \times 10^3\ N/mm^2$

Elongation in extension $\longrightarrow$ +ve

Elongation in Compressive $\longrightarrow$ -ve

Total elongation ($\Delta L_T$)

$$= \frac{AB}{\Delta L_1} - \frac{BC}{\Delta L_2} + \frac{CD}{\Delta L_3}$$

$$= \frac{P_1 L_1}{A_1 E_1} - \frac{P_2 L_2}{A_2 E_2} + \frac{P_3 L_3}{A_3 L_3}$$

$A_1 = A_2 = A_3 = (A) = 25 mm^2$

$E_1 = E_2 = E_3 = (E) = 200 \times 10^3 N/mm^2$

$$\Delta L_T = \frac{P_1 L_1}{AE} - \frac{P_2 L_2}{AE} + \frac{P_3 L_3}{AE}$$

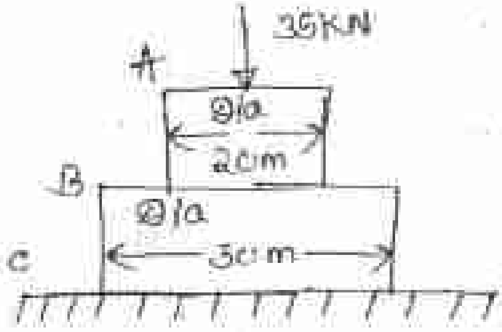
$$= \frac{1}{AE}\left[P_1 L_1 - P_2 L_2 + P_3 L_3\right]$$

$$= \frac{1}{25 \times 200 \times 10^3} \left[ (100 \times 500) - (150 \times 800) + (50 \times 400) \right]$$

$\Delta L_T = -0.01\,mm$ (Composite)

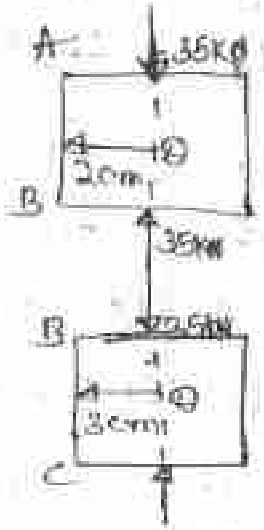
$\therefore$ -ve singn indicate the bar is subjected to compressive.

<u>Q2</u> A stepped bar as shown in the figure is subjected to an axially applied load of 35 KN. find the maximum and minimum stress developed.




16 oct 2020

<u>Soln</u> <u>F.B.D</u>




$$\text{Stress } (\sigma) = \frac{\text{Resisting Force } (F)}{\text{Area}}$$

Resisting is equal to applied force

$F = P \quad P \rightarrow$ (applied force)

$$\sigma = \frac{P_{applied}}{Area}$$

$\sigma \propto P_{applied}$

$\sigma \propto \frac{1}{A}$

<u>Step-1</u> $\boxed{\pi = 22/7}$

<u>Data given</u>

Dia of bar 'AB' $= d_1 \; 2\,cm = 20\,mm$

Dia of bar 'BC' $= d_2 \; 3\,cm = 30\,cm$

Area of bar $AB (A_1) = \frac{\pi}{4} \times 20^2$

$A_1 = 2270\,mm^2$

Area of bar $BC (A_2) = \frac{\pi}{4} \times 30^2$

$A_2 = 5107.5\,mm^2$

$A_1 < A_2$

So AB is subjected of max$^m$ stress where as bar BC is subjected to min$^m$ stress.

$$\sigma_{AB} = \frac{\text{Resisting force}}{Area} = \frac{F_1}{A_1}$$

$F_1 = 35\,kN = 35 \times 10^3\,N$

$$\sigma_{AB} = \frac{35 \times 10^3}{2278} = 15.48\,N/mm^2$$

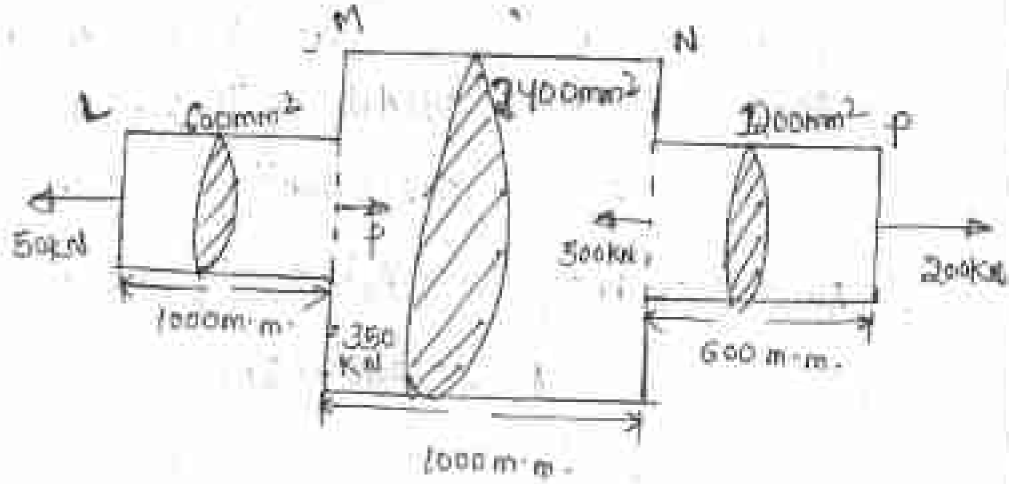
$$\sigma_{BC} = \frac{\text{Resisting force}}{Area} = \frac{F_2}{A_2}$$

$$= \frac{35 \times 10^3}{5107.5} = 6.852\,N/mm^2$$

$\sigma_{max}(AB) = 15.48\ N/mm^2$

$\sigma_{min}(BC) = 6.852\ N/mm^2$ Ans

Q3. A member LMNP is subjected to as shown in the figure. calculate
(i) force necessary for equilibrium
(ii) Total elongation in the bar
(iii) Take $E = 200\ GN/m^2$




Soln Condition of equillibrium

$\Sigma f_x = 0$

$\Sigma f_y = 0$

$\Sigma M_z = 0$

Total Right word force in x direction
= Total Left word force in x-direction

$\Rightarrow P + 200\,kN = 500\,kN + 50\,kN$

$\Rightarrow P + 200\,kN = 550\,kN$

$\boxed{P = 550\,kN - 200\,kN = 350\,kN}$

Total elongation in composite bar

$= \Delta L_T = \Sigma [\Delta L_1 + \Delta L_2 \cdots \Delta L_n]$

(According to principle of super position)

19 oct 2020

Step - 1

Data given :-

Area of segement ① $= (A_1) = 600\ mm^2$

Area of segement ② $= (A_2) = 2400\ mm^2$

Area of segement ③ $= (A_3) = 1200\ mm^2$

Length of segement ① $= (L_1) = 1000\ m.m.$

Length of segement ② $= (L_2) = 1000\ mm.$

Length of segement ③ $= (L_3) = 600\ mm.$

E for the all the segement

$= 200\ GN/m^2$

$= 200 \times 10^9\ N/m^2$

$1 m = 1000\ m.m.$

$1 m^2 = 1000 \times 1000\ mm^2$

$= 10^6\ mm^2$

$E = \frac{200 \times 10^9\ N}{10^6\ m.m^2}$ $\quad \frac{a^x}{a^y} = a^{x-y}$

$200 \times 10^{(9-6)}\ N/mm^2$

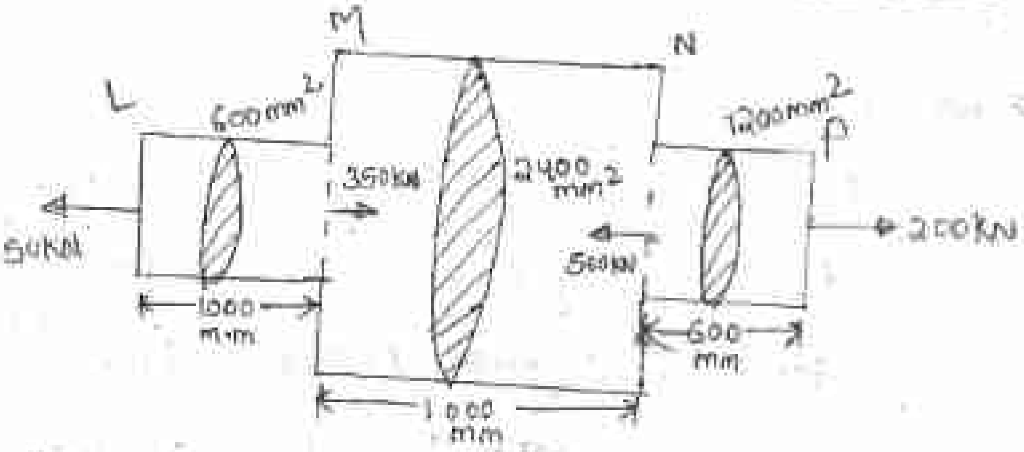
$= 200 \times 10^3\ N/mm^2$

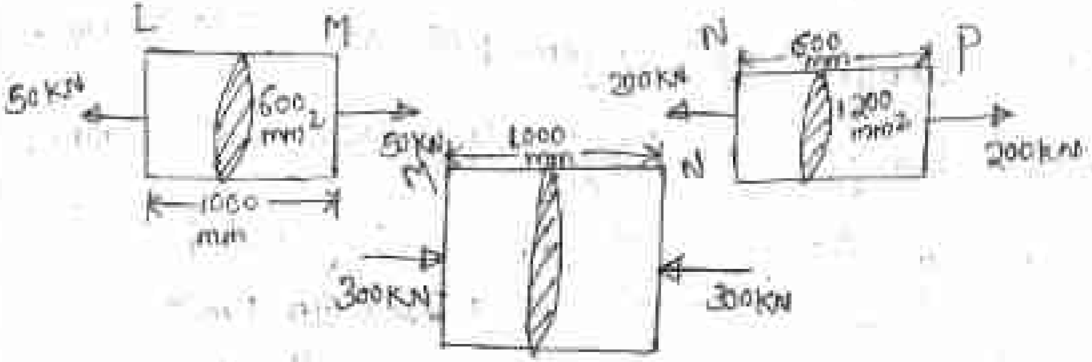
$\Delta L_T = \Sigma [\Delta L_1 + \Delta L_2 \cdots \Delta L_n]$

$$\Rightarrow \Delta L_T = \left[\Delta L_1 + \Delta L_2 + \Delta L_3\right]$$

$$\Rightarrow \Delta L_T = \left[\frac{P_1 L_1}{A_1 E} + \frac{P_2 L_2}{A_2 E} + \frac{P_3 L_3}{A_3 E}\right]$$

Step-II F.B.D of different section




Load on segement (I) $(P_1) = 50\,kN = 50\times10^3\,N$

Load on segement (II) $(P_2) = 300\,kN = 300\times10^3\,N$

Load on segement (III) $(P_3) = 200\,kN = 200\times10^3\,N$

extention → +ve sign

Compaction → -ve sign

$$\underset{(LMNP)}{\Delta L_T} = \underset{[LM]}{\Delta L_1} - \underset{[MN]}{\Delta L_2} + \underset{[NP]}{\Delta L_3}$$

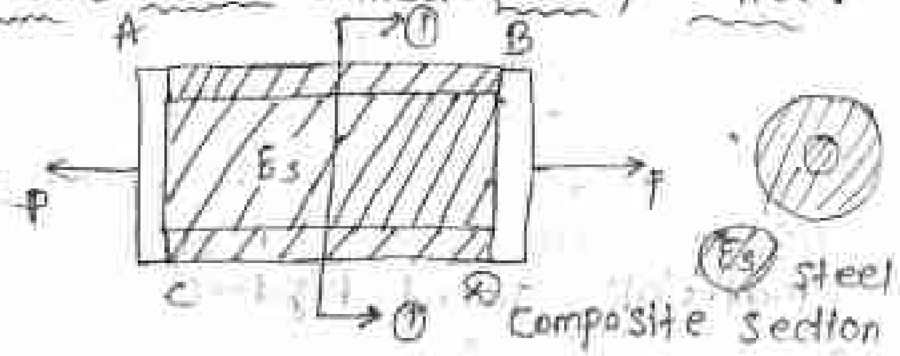
$$= \frac{P_1 L_1}{A_1 E} - \frac{P_2 L_2}{A_2 E} + \frac{P_3 L_3}{A_3 E}$$

$$= \frac{1}{E}\left[\frac{P_1 L_1}{A_1} - \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3}\right]$$

$$= \frac{1}{200\times10^3}\left[\frac{50\times10^3\times100}{600} - \frac{300\times10^3\times1000}{2400} + \frac{200\times10^3\times600}{1200}\right]$$

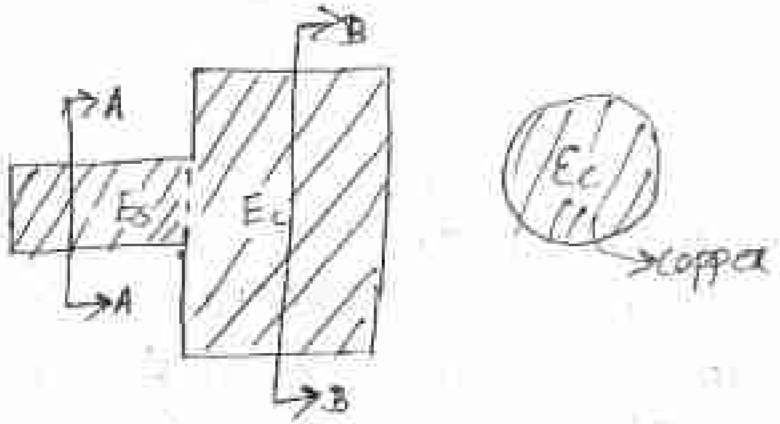
$\Delta L_T = 0.291\,mm$

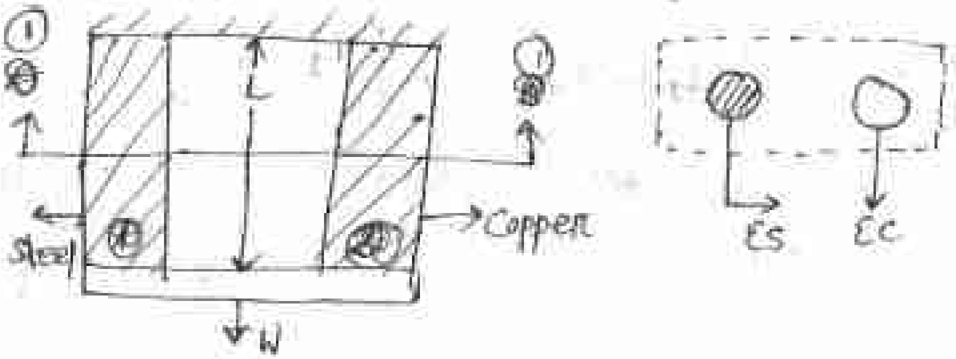
Members are Connected in parallel :-




Composite section

They are connected in parallel.




$W = W_c + W_s$, $\delta l$ steel $= \delta l$ copper

⇒ The applied load is shared among the member $\boxed{W = W_c + W_s}$

⇒ Deformation of each member will be equal

$$\delta l_1 = \delta l_2$$

20 oct 2020

Analysis of bars of composite section:-

⇒ A composite bar may be defined as the bar is made up of two or more different materials joined together.

⇒

⇒ for composite bar the following two points are important.

① The extension or compression is equal i.e strain in each bar is equal

$\boxed{\delta l_1 = \delta l_2}$ , $\boxed{e_1 = e_2}$

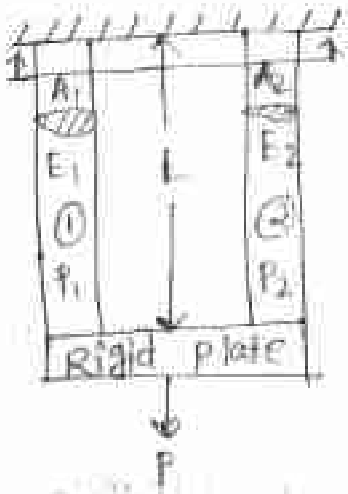
② The total external load on the composite bar is equal to the sum of the loads carried by each different material.

$W = W_1 + W_2$

⇒ Consider a composite bar made up of two different materials.

Let $P \rightarrow$ Total load on composite bar

$L \rightarrow$ length of composite bar and also lengths of bars of different




$A_1$ & $A_2 \rightarrow$ C/S area of bar ① and ② respectively.

$E_1$ & $E_2 \rightarrow$ Young's modulus of bar ① and ②

$P_1$ & $P_2 \rightarrow$ Load shared by bar ① and ②

$\sigma_1$ & $\sigma_2 \rightarrow$ stress induced in bar ① and bar ② respectively.

Total load (P) = $P_1 + P_2$

(on composite bar) $P = P_1 + P_2$

Stress in bar ① = $\frac{P_1}{A_1}$

$$\Rightarrow \sigma_1 = \frac{P_1}{A_1} \Rightarrow P_1 = \sigma_1 A_1 \quad \text{—— (ii)}$$

Similarly stress bar ② ($\sigma_2$) = $\frac{P_2}{A_2} \Rightarrow P_2 = \sigma_2 A_2$ —— (iii)

Put eq (ii) & (iii) in eqn ①

~~Strain in member ① = Strain in member II~~

$$P = P_1 + P_2 = \sigma_1 A_1 + \sigma_2 A_2.$$

$$P = \sigma_1 A_1 + \sigma_2 A_2$$

(ii) Strain in member ① = Strain in member II

$$\Rightarrow \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

$$\Rightarrow \sigma_1 = \frac{E_1}{E_2} \sigma_2$$

$\boxed{\frac{E_1}{E_2} = m}$ (modular ratio)

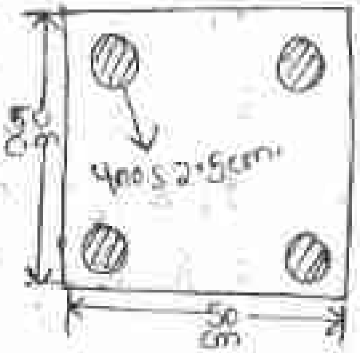
Q1 A reinforced concrete column 50cm×50 cm in section is reinforced with 4 nos of steel bar of dia 2.5 cm one in each corner. The column is carrying a load 200 tonnes. find the stress in concrete and steel. Take $E_s = 2.1\times10^6$ kg/cm$^2$

$E_c = 0.14\times10^6$ kg/cm$^2$

Data given :-

Total load on column = 200 tonnes

(P) $= 200\times10^3$ kg




C/S area of the column

Ag (gross Area) = 50cm×50

= 2500cm$^2$

C/S area of steel $= 4\times\frac{\pi}{4}\times d^2$

$= \pi\times2.5^2 = 19.634$ cm$^2$

C/C Area of Concrete = Ag − As

$A_c = 2500 - 19.634$

$= 2480.366$ cm$^2$

Total load = Load on steel + load on concrete

$\Rightarrow P = P_s + P_c$

$\Rightarrow P = \sigma_s A_s + \sigma_c A_c$

$\Rightarrow 200\times10^3 = \sigma_s\times19.634 + \sigma_c\times(2480.366)$ — eq$^n$ (1)

(ii) Strain in steel = strain in concrete

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \frac{E_s}{E_c}\sigma_c \qquad \frac{E_s}{E_c} = \frac{2{\cdot}1\times10^6\ kg/cm^2}{0{\cdot}14\times10^6\ kg/cm^2}$$

$$= 15$$

$$\Rightarrow \sigma_s = 15\sigma_c$$

Put the value of eq (ii) in eq$^n$ (i)

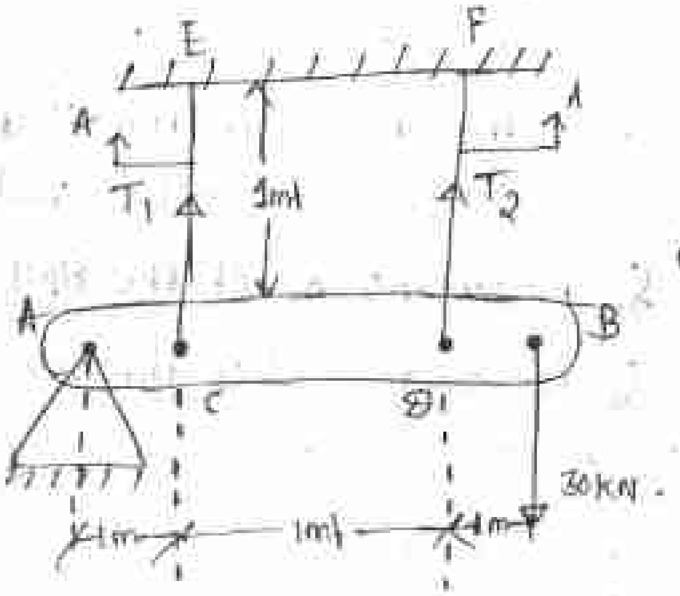
$$\Rightarrow 200\times10^3 = 15\sigma_c(19{\cdot}634) + \sigma_c(2480{\cdot}366)$$

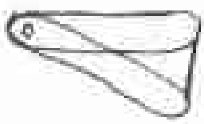
$$\Rightarrow \sigma_c = 72\ kg/cm^2$$

$$\sigma_s = 15\sigma_c = 15\times72 = 1080\ kg/cm^2$$

21 oct 2020

**Q2** A rigid bar ABC is hinged at 'A' and supported in a horizontal position by two identical steel wires as shown in figure. A vertical load of 30 kN is applied at 'B' find the tensile forces $T_1$ and $T_2$ induce in the wires by vertical load.



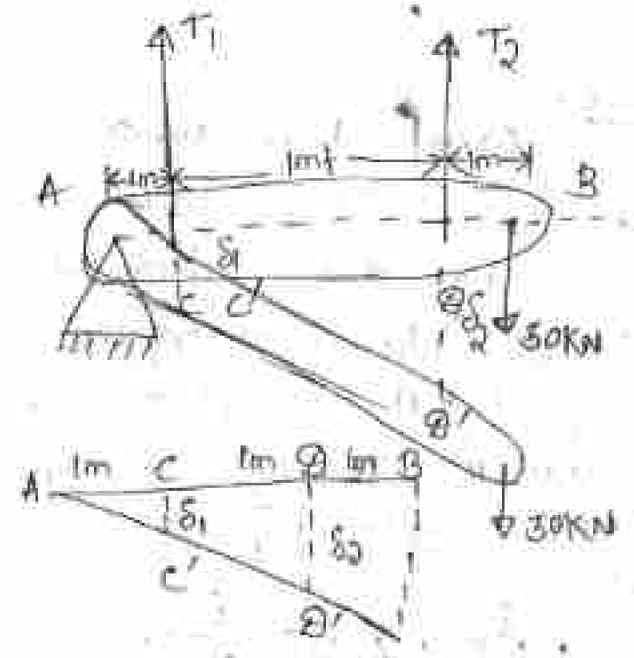



Rigid bar means the bar which remain straight.

Two Identical steel wires means the area of cross-section, length and the value of 'E' for both the wire are same.

$(A_1 = A_2,\ l_1 = l_2 = 1\ (1m)\ E_1 = E_2)$

F.B.D




Solⁿ

$T_1$ be the tension in the first wire.

$T_2$ be the tension in the 2nd wire.

$\delta_1$ be extension of the first wire.

$\delta_2$ be extension of the 2nd wire.

$\Delta ACC'$ and $\Delta ADD'$

$\Delta ACC' \cong \Delta ADD'$

$$\frac{\delta_1}{\delta_2} = \frac{AC}{AD}$$

$\Rightarrow \frac{\delta_1}{\delta_2} = \frac{1}{2}$

$\Rightarrow \boxed{\delta_2 = 2\delta_1}$ —— (i)

$\delta_1 = \frac{P_1 L}{A_1 E_1}$ , $\delta_2 = \frac{P_2 L}{A_2 E_2}$

Put the values of $\delta_1$ & $\delta_2$ in eq$^n$ (i)

$\Rightarrow \frac{P_2 L}{A_2 E_2} = 2 \times \left[\frac{P_1 L_1}{A_1 E_1}\right]$

$\Rightarrow \frac{T_2 L}{A_2 E_2} = 2 \times \left[\frac{T_1 L}{A_1 E_1}\right]$

Replace $P_1$ & $P_2$ by $T_1$ & $T_2$ (Resisting force)

$A_1 = A_2 = A$ , $L_1 = L_2 = L$ , $E_1 = E_2 = E$

$\Rightarrow \frac{T_2 \cancel{L}}{\cancel{AE}} = 2 \times \left[\frac{T_1 \cancel{L}}{\cancel{AE}}\right]$

$\Rightarrow \boxed{T_2 = 2T_1}$ —— (ii)

Taking moment at 'A' , $\Sigma M_A = 0$

~~Total~~ T.A.M = T.C.M at 'A'

$\Rightarrow T_1 \times 1 + T_2 \times 2 = 30 \times 3$

$\Rightarrow T_1 + 2T_2 = 90$ —— (iii)

$\boxed{T_2 = 2T_1}$ —— (ii)

Put the value of eq$^n$ (ii) & (iii)

$\Rightarrow T_1 + 2(2T_1) = 90$

$\Rightarrow T_1 + 4T_1 = 90$

$\Rightarrow 5T_1 = 90$

$\Rightarrow T_1 = \frac{90}{5} = 18\,KN$

put the value $T_1$ in eq (ii)

$T_2 = 2T_1$

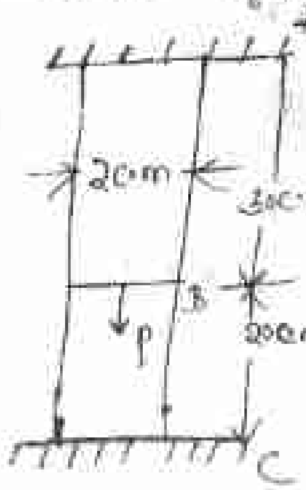
$\Rightarrow T_2 = 2\times18 = 36\,KN$

$T_1 = 18KN$ , $T_2 = 36\,KN$ Ans

3 NOV 2020

Problem - 1

A square bar of 2cm side is held between two rigid plates and loaded by an axial force of 'p' equal to 30 tonnes as shown in the figure find the reactions at the ends 'A' and 'c' and the extension of the portion 'AB' Take $E = 2\times10^6\ kg/cm^2$



Sol^n

Step-(i)

Data given:-

Side of the bar $(a) = 2\,cm$.

Force on the bar $(P) = 30\,ton$
$= 30\times10^3\,kg$

Length of bar (AB) $l_{AB} = 30\,cm$.

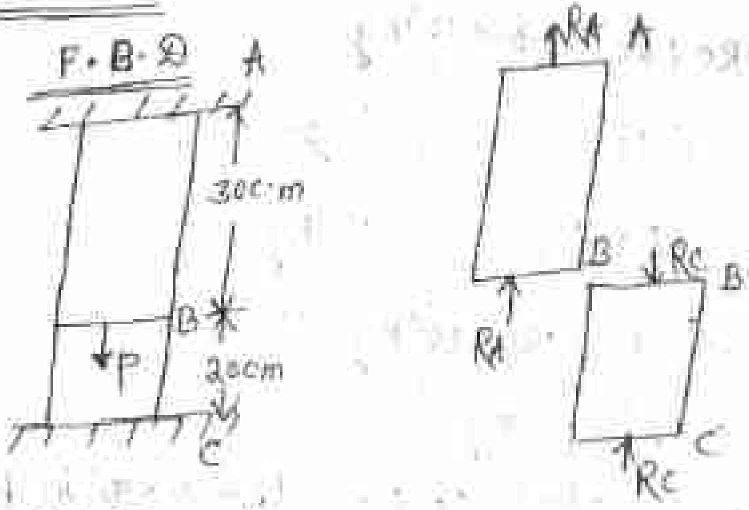
Length of bar (BC) $l_{BC} = 20\,cm$

Youngs modulus $(E) = 2\times10^6\,kg/cm^2$

Area of the bar $= a^2 = 2^2\,cm = 4\,cm^2$

Step-(ii)

F.B.D




$R_A + R_C = P$

Step-(iii)

$R_A + R_C = P$ —— ①

$R_A + R_C = 30\times10^3\,kg$ —— ①

Elongation in bar AB = Compression in the bar BC.

$\Rightarrow \Delta l_{AB} = \Delta l_{BC}$

$\Rightarrow \dfrac{P_{AB}l_{AB}}{A_{AB}E} = \dfrac{P_{BC}l_{BC}}{A_{BC}E}$

$$\Rightarrow \frac{P_{AB}L_{AB}}{A_{AB}\times E} = \frac{P_{BC}l_{BC}}{A_{BC}\times E}$$

$$\Rightarrow P_{AB}L_{AB} = P_{BC}\times L_{BC}$$

$$\Rightarrow R_A L_{AB} = R_C l_{BC}$$

$$\Rightarrow R_A 30 = R_C 20$$

$$\Rightarrow R_A = \frac{20}{30} R_C$$

$$\boxed{R_A = \frac{2}{3} R_C} \text{ — (ii) eq}^n$$

Put the value of $R_A = \frac{2}{3} R_C$ in eq$^n$ (i)

$$R_A + R_C = 30\times 10^3 \text{ kg}$$

$$\Rightarrow \frac{2}{3} R_C + R_C = 30\times 10^3 \text{ kg}$$

$$\Rightarrow R_C \left(1+\frac{2}{3}\right) = 30\times 10^3 \text{ kg}$$

$$\Rightarrow R_C \left(\frac{3+2}{3}\right) = 30\times 10^3 \text{ kg}$$

$$\Rightarrow R_C \left(\frac{5}{3}\right) = 30\times 10^3 \text{ kg}$$

$$R_C = 18{,}000 \text{ kg}.$$

Put the value of $R_C = 18000$ kg in the eq$^n$ (i).

$$R_A = \frac{2}{3} R_C$$

$$= \frac{2}{3}\times 18000 = 12000 \text{ kg}$$

(ii) Elongation in the bar 'AB'

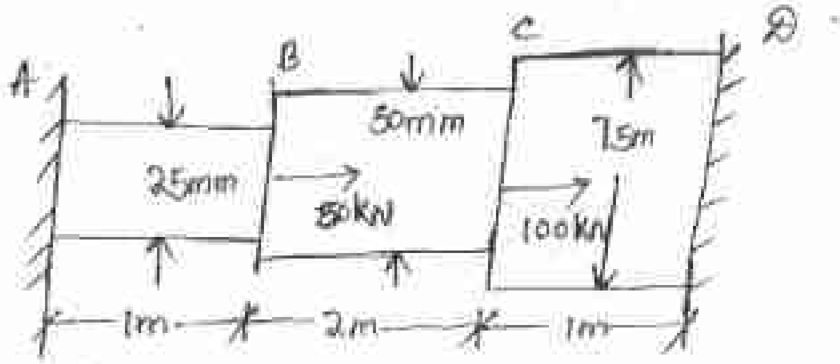
$$\delta l_{AB} = \frac{P_{AB}L_{AB}}{A_{AB}\times E}$$

$$= \frac{R_A \times 30 \text{ cm}}{4 \text{ cm}^2 \times 2\times 10^6 \text{ kg/cm}^2}$$

$$\delta l_{AB} = \frac{12000 \times 30}{4 \times 2 \times 10^{6}} = 0.045 \text{ cm} \quad \underline{\text{Ans}}$$

Problem -2

A circular steel bar ABCD rigidly fixed with at 'A' and 'D' is subjected to axial loads of 50 kN and 100 kN . at 'B' and 'C' as shown in the figure . find the loads shared by each part of the bar and the displacement of the points 'B' and 'C' Take "$E_s$" = 207 kN/mm²

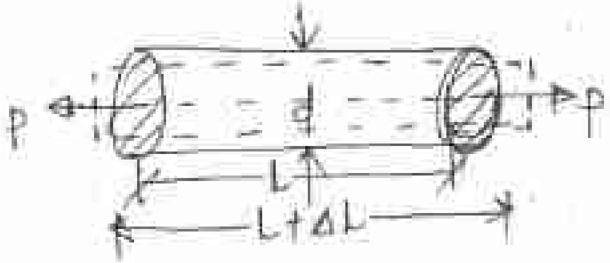



4 nove 2020

4 nov 2020

## Poisson's Ratio :-

It is defined as the ratio of lateral strain to Longitudinal strain.




**Mathematically :-**

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$= \frac{\frac{\Delta D}{D}\text{(circular)}\ \frac{\Delta d}{d}\ \text{or}\ \frac{\Delta b}{b}\text{(rectangle)}}{\frac{\Delta L}{L}}$$

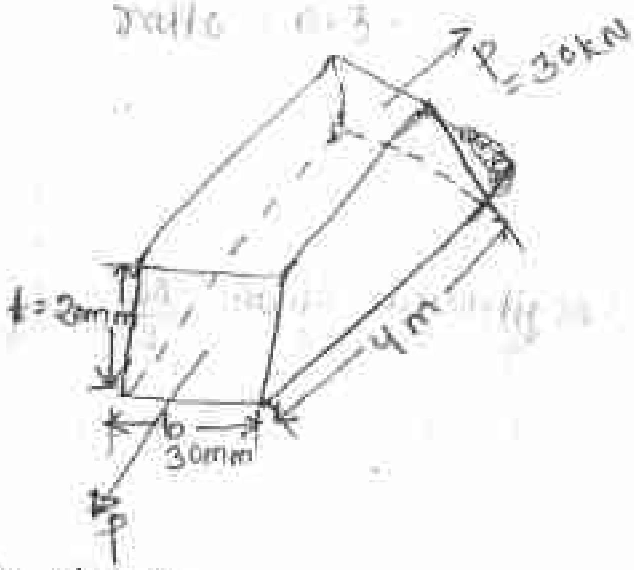
→ lateral strain = $\mu$ × Longitudinal strain

→ The value of $\mu$ varies from 0.25 to 0.33.

→ It is a demention less quantity.

Problem-1 Determine the change in length breadth and thickness of a steel bar which is 4m long 30mm breadth & 20mm thick and it is subjected to an axial pull of 30kN in the direction of its length take $E = 2\times10^5 \, N/mm^2$ and poisson ratio 0.3.

Sol^n




Data given :-

Length of the bar (L) = 4m.

width of the bar (b) = 30mm

Thickness of the bar (t) = 20mm.

Axial pull (P) = 30kN / $30\times10^3$ kN

Young's modulus (E) = $2\times10^5$ N/mm$^2$

poisson's ratio ($\mu$) = 0.3

Area of cross section (A) = 30mm × 20mm
= 600 mm$^2$

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\Rightarrow \mu = \frac{\frac{\Delta b}{b} \text{ and } \frac{\Delta t}{t}}{\frac{\Delta L}{L}}$$

$= \Delta L = \frac{PL}{AE}$

<u>Step - II</u> Change in length

$$\Delta L = \frac{PL}{AE}$$

$$= \frac{30 \times 10^3 \times 4000}{600 \times 2 \times 10^5 \text{ N/mm}}$$

$$\Delta L = 1.0 \text{ mm}$$

<u>Step - III</u>

$$\text{poisson ratio}(\mu) = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\text{Longitudinal strain } \frac{\Delta L}{L} = \frac{1}{4000}$$

$\Rightarrow$ Lateral strain $= \mu \times$ Longitudinal strain

$$\Rightarrow \frac{\Delta b}{b} \text{ and } \frac{\Delta t}{t} = 0.3 \times \frac{1}{4000}$$

$$\frac{\Delta b}{b} = 0.3 \times \frac{1}{4000}$$

$$\Delta b = 0.3 \times \frac{1}{4000} \times 30$$

$$\Delta b = 0.00225 \text{ mm}$$

$$\Delta t = 0.3 \times \frac{1}{4000}$$

$$= \Delta t = 0.3 \times \frac{1}{4000} \times 20 = 0.0015 \text{ mm}$$

<u>Ans</u>

Hookes law:-

It states that within elastic limit stress is directly proportional to the strain.

Mathmetically

$$\text{Stress} \propto \text{Strain}$$

Young's modulus

$$\text{Stress} = \text{constant} \times \text{strain}$$

Modulus of rigidity

$$\text{Constant} = \frac{\text{Stress}}{\text{Strain}}$$

Bulk modulus of elasticity

Young's modulus (E)

$$= \frac{\text{tensile stress or compressive stress}}{\text{tensile strain or compressive strain}}$$

$$E = \frac{\sigma_N}{\text{longitudinal and lateral strain}}$$

Modulus of rigidity (C, or G or N)

→ It is the ratio betn shear stress to shear strain.

→ It is denoted by C, N or G.

→ Mathmetically (G) $= \frac{\tau}{\phi}$

6 NOV 2020

## Bulk modulus (K) :-

- It is the ratio betn normal stress and volumetric strain.
- It is denoted by 'K'
- Mathmetically $K = \frac{\text{Normal stress}}{\text{Volumetric Strain}}$

$$= \frac{\sigma_N}{e_V}$$

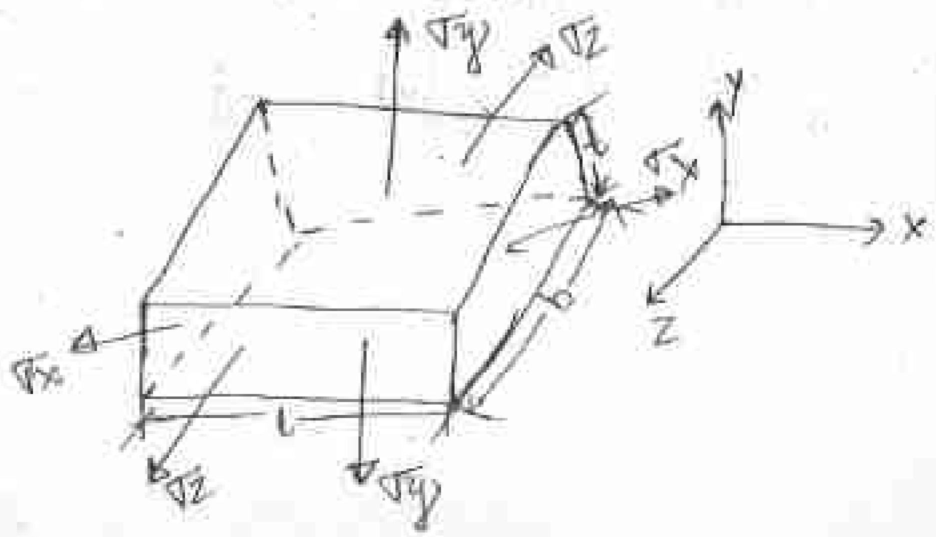
$(\sigma_x = \sigma_y = \sigma_z = \sigma) \rightarrow$ bulk stress.

## Volumetric Strain :-

- It is the ratio betn change in volume to it's original volume.
- It is denoted by $e_v$ or $\epsilon_v$
- Mathmetically $\epsilon_v = \frac{\Delta V}{V}$

$$\Delta V = \frac{\text{Final volume} - \text{Initial volume}}{\text{Initial volume}}$$

IMP * Volumetric Strain of a rectangular body subjected to three mutually perpendicular forces of stress :-



Consider a rectangular body subjected to direct tensile stresses (+ve) along three perpendicular axes as shown in the fig.

$\mu \rightarrow$ poisson Ratio

Let $\sigma_x \longrightarrow$ stress in x-x direction.

$\sigma_y \longrightarrow$ stress in y-y direction.

$\sigma_z \longrightarrow$ stress in z-z direction.

$E \longrightarrow$ Young's modulus of material

Volumetric Strain ($\varepsilon_v$) (ipselon v)

$$= \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\boxed{\left[ E = \frac{\sigma}{\varepsilon} \right] \Rightarrow \varepsilon = \frac{\sigma}{E}}$$

Strain in x-x direction $\varepsilon_x = \frac{\sigma_x}{E}$

strain in y-y direction $\varepsilon_y = \frac{\sigma_y}{E}$

Strain in z-z direction $\varepsilon_z = \frac{\sigma_z}{E}$

A little consideration will take that when the stress applied direction is subjected to elongation where as the opposite two direction subjected to compression.

Actual $\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

Lateral strain = $\mu \times$ longitudinal strain

$$\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} - \mu\frac{\sigma_y}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$= \left(\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}\right) - \left(\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E}\right) + \left(\frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}\right)$$

$$= \left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right)\left(-2\mu\frac{\sigma_x}{E} - 2\mu\frac{\sigma_y}{E} - 2\mu\frac{\sigma_z}{E}\right)$$

$$= \left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right)\left[-2\mu\left(\frac{\sigma_x}{E} + \frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right)\right]$$

$$= \left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right)\left[-2\mu\left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right)\right]$$

I.M.T $\varepsilon_v = \left[\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right]\left[1 - 2\mu\right]$ (uniaxial)

$\varepsilon_v = \frac{\sigma_x}{E}(1-2\mu)$ (biaxial) if $\sigma_y = 0$, $\sigma_z = 0$

$\varepsilon_z = \frac{\sigma_x + \sigma_y}{E}(1-2\mu)$ if $\sigma_z = 0$ (triaxial)

$\varepsilon_v = \frac{\sigma_x}{E}(1-2\mu)$

Problem - I

A steel bar 2m long 20mm wide and 15mm thick is subjected to a tensile load of 30kN find, increase in volume is poisson's reation ($\mu$) = 0.25 and young's modulus (E) = 200 Gpa

$$\varepsilon_v = \frac{\sigma_x + \cancel{\sigma_y}^{0} + \cancel{\sigma_z}^{0}}{E}(1-2\mu)$$

$$\varepsilon_v = \frac{\sigma_x}{E}(1-2\mu) \quad \cancel{\text{axial}}$$

$$\frac{\Delta v}{v} = \frac{\sigma_x}{E}(1-2\mu)$$

$\Delta v = ?$

Sol^n

Data given :-

Length of steel bar (L) = 2m.

Width (b) = 20mm

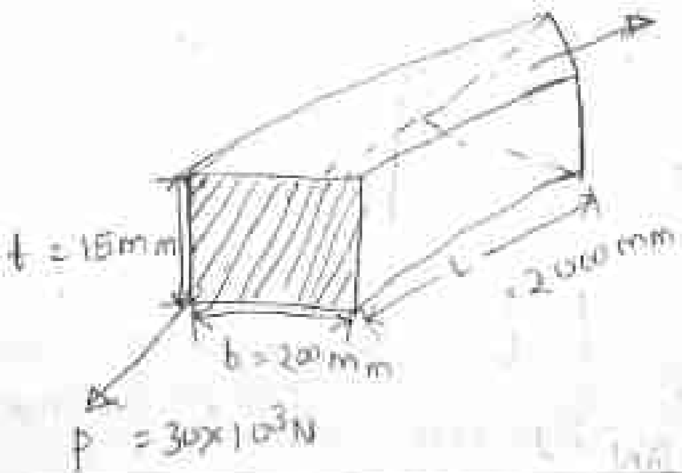
Thickness (t) = 15mm

tensile load (P) = 10kN

poisson's ratio ($\mu$) = 0.25

7NOV2020 Young's modulus (E) = 200 Gpa

$= 200 \times 10^3\ N/mm^2$.




$v = L \times b \times t$

$= 2000 \times 20 \times 15$

$= 600 \times 10^3$

$$\varepsilon_v = \frac{\sigma_x}{E}(1-2\mu)$$

$$\sigma_x = \frac{P}{A} = \frac{P}{b \times t} \quad \text{——— (1)}$$

$$\varepsilon_v = \frac{\sigma_x}{E}(1-2\mu)$$

$$= \frac{P}{btE}(1-2\mu)$$

$$\therefore \sigma_x = \frac{P}{bt}$$

$$\varepsilon_v = \frac{30 \times 10^3}{20 \times 15 \times 200 \times 10^3}(1-2 \times 0.25)$$

$$= 0.00025$$

$$\varepsilon_v = \frac{\Delta V}{V} \Rightarrow \frac{\Delta V}{V} = 0.00025$$

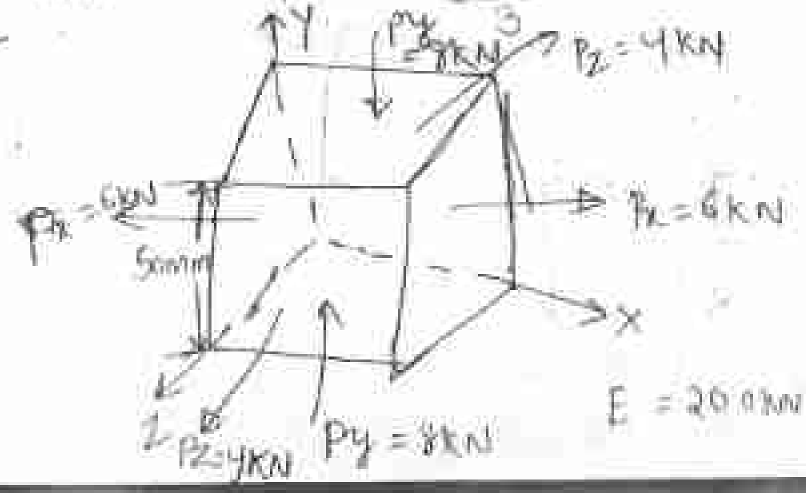
$$\Rightarrow \Delta V = 0.00025 \times V$$

$$= 0.00025 \times (600 \times 10^3)$$

Problem - 2

A steel block, cube of 50mm wide is subjected to a force of 6 kN (Tensile) 8kN (compression) and 4kN (Tensile) along x, y, and z direction respectively. Determine the change in volume of the block take $E = 200 kN/mm^2$ and $m = \frac{10}{3}$

Soln:-




$E = 200 kN/mm^2$

Step-I

Side of the cube (a) = 50mm.

Force in x-x direction = ($P_x$) = 6KN

= $6\times10^3$ N (Tension)

Force in y-y direction ($P_y$) = 8KN

= $8\times10^3$ N

(Compressive)

Force in z-z direction ($P_z$) = 4KN = $4\times10^3$ N

(Tensile)

$E = 200\ \text{KN/mm}^2 = 200\times10^3\ \text{N/mm}^2$

$m = \frac{10}{3}$

$\Rightarrow \mu = \frac{1}{m} = \frac{3}{10}$

change in volume ($\Delta V$)

$$= \left[\left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right)(1-2\mu)\right]\times V$$

Step-II original volume of steel cube

$V = a^3$

$= 50^3 = 125\times10^3\ \text{mm}^3$.

Stress in x-x direction

$$\sigma_x = \frac{P_x}{A} = \frac{6\times10^3}{50\times50} = 2.4\ \text{N/mm}^2$$

(Tension)

Stress in y-y direction

$$\sigma_y = \frac{P_y}{A} = \frac{8\times10^3}{50\times50} = 3.2\ \text{N/mm}^2$$

Stress in z-z direction

$$\sigma_z = \frac{P_z}{A} = \frac{4\times10^3}{50\times50} = 1.6\ \text{N/mm}^2$$

Step - III

strain in x-x direction $= \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$

$$= \frac{2.4}{E} - \frac{3.2\times3}{E} - \frac{3}{10}\times\frac{1.6}{E}$$

$$= \frac{2.88}{E}$$

strain in y-y direction

$$\epsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

$$= \frac{-3.2}{E} - \frac{3}{10}\times\frac{2.4}{E} - \frac{3}{10}\times\frac{1.6}{E}$$

$$\cancel{= \frac{1}{E} - 3.2 - \frac{3\times2.4}{10\,E} - \frac{3}{10}\times\frac{1.6}{10\,E}}$$

$$= \frac{1}{E}\left[-3.2 - \frac{3\times2.4}{10} - \frac{3\times1.6}{10}\right]$$

$$= \frac{-4.4}{E}$$

Strain in z-z direction

$$\epsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{(-\sigma_y)}{E}$$

$$= \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} + \mu\frac{\sigma_y}{E}$$

$$= \frac{1.6}{E} - \frac{3}{10}\times\frac{2.4}{E} + \frac{3}{10}\times\frac{3.2}{E}$$

$$= \frac{1}{E}\left[1.6 - \frac{3\times2.4}{10} + \frac{3\times3.2}{10}\right]$$

$$= \frac{1}{E}\times1.84$$

$$= \frac{1.84}{E}$$

Step - (IV)

volumetric strain

$$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= \frac{2.88}{E} - \frac{4.4}{E} + \frac{1.84}{E}$$

$$= \frac{1}{E}\left[2.88 - 4.4 + 1.84\right]$$

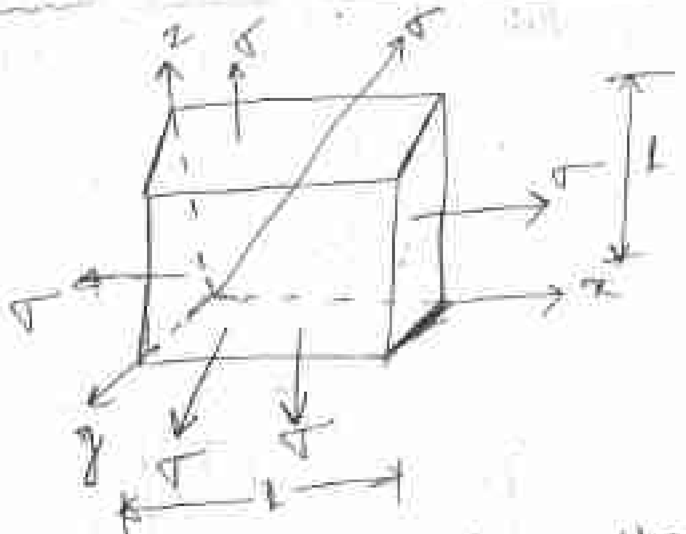
$$\Rightarrow \varepsilon_V = \frac{1}{E}\left[0.32\right]$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{0.32}{E}$$

change in volume $= \Delta V = \frac{0.32}{E} \times V = \frac{0.32}{200 \times 10^3}$

$\Delta V = 0.2 mm^3$ Ans

V.V.I Relation betn Bulk modulus(k) and Young's modulus (E) :-




Consider a cube whose sides are 'l'
Let the cube is subjected to three mutual perpendicular stresses (Tensile) of equal in tensility

let σ → stress on the forces

l → length / sides of cube

E → Young's modulus

x → poisson ratio

We known that volumetric strain
$$\varepsilon_v = \varepsilon_x - \varepsilon_y - \varepsilon_z$$

Strain in x-x direction
$$\varepsilon_x = \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E}$$

Strain in y-y direction
$$\varepsilon_y = \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E}$$

Strain in z-z direction.
$$\varepsilon_z = \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E}$$

$$\varepsilon_v = \left(\frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E}\right) + \left(\frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E}\right)$$
$$+ \left(\frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E}\right)$$
$$= \frac{3\sigma}{E}\left[(1-2\mu)\right]$$

Bulk modulus (K) = $\dfrac{\text{Normal stress}}{\text{volumetric strain}}$

$$= \frac{\sigma}{\varepsilon_v}$$

$$\Rightarrow K = \frac{\sigma}{\frac{3\sigma}{E}(1-2\mu)}$$

$$\Rightarrow K = \frac{E}{3}(1-2\mu)$$

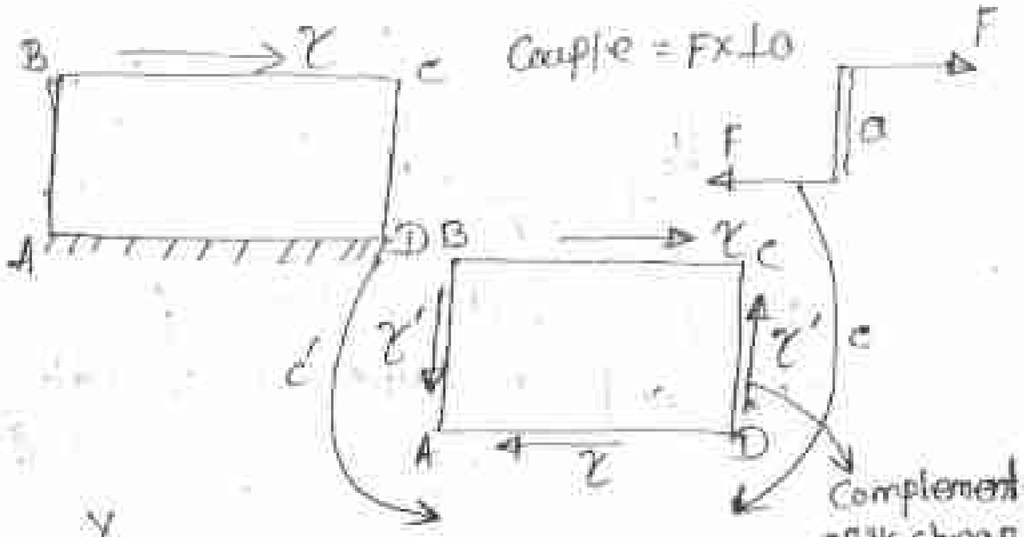
$$\Rightarrow \boxed{E = 3K(1-2\mu)} \quad (\text{v.v.I})$$

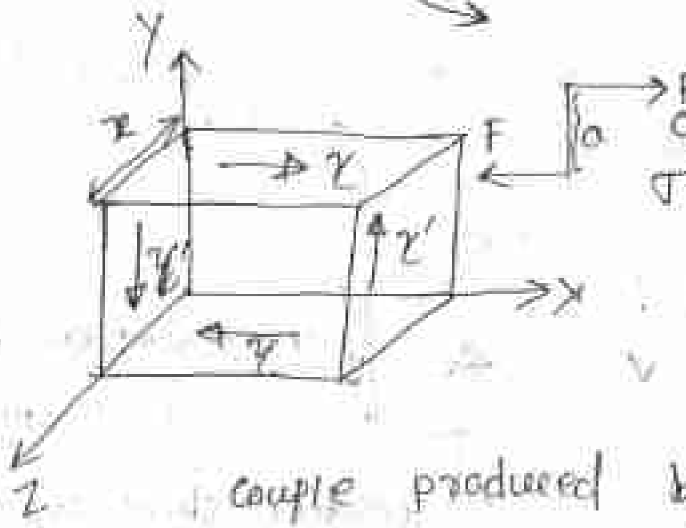
19 Nov 2020

Principle of shear stress :-

It states that "A shear stress across a plane is always accompanied by a balancing shear stress across the plane and normal to it.

Proof :- (i) Consider a rectangular block ABCD of thickness 't' is subjected to a shear stress $\tau$ (Tau) on face BC, and the face AD is fixed.




Couple = F×a

$\sigma = \frac{F}{A}$

$F = \sigma A$

$\tau = \frac{F}{A}$

$F' = \tau A$

Couple produced by $\tau$

$= (\underbrace{\tau \times BC \times t}_{\text{Force}}) \times AB$

Couple produced by $\tau'$ (anticlockwise)

$c' = (\underbrace{\tau' \times AB \times t}_{\text{Force}}) \times BC$

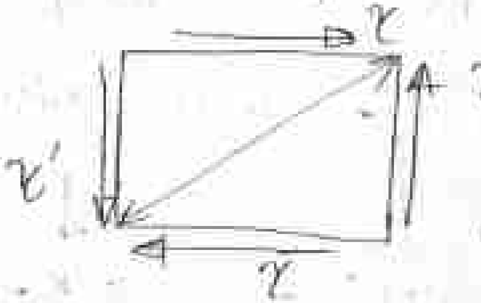
For equillibrium of element ABCD

$c' = c$

$\Rightarrow \tau' \times AB \times t \times BC = \tau \times BC \times t \times AB$

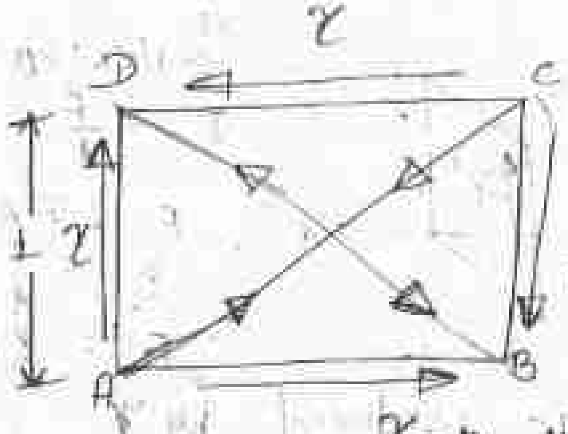
$= \tau = \tau'$

Every shear stress is accompained by an equal complementary shear stress on a plane at right angle.



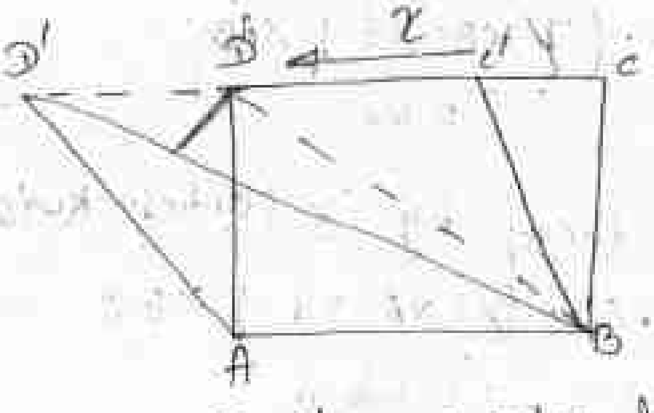

$\boxed{\tau = \tau'}$

- Relation betn modulus of elasticity (E) and modulus of rigidity (C/G) :-




τ ⟷ Complementary shear stress

τ : Resultive shear stress




Consider a cube length (l) subjected to shear stress τ as shown in the figure.

THEORY SUBJECT - Structural Mechanics (TH-1)

→ A little consideration will show that due to these

→ stress the cube is subjected to some distruction.

→ Such that the diagonal BD will be elongated and the diagonal 'AC' will be shortend.

→ Let the shear stress $\tau$ cause the shearstrain ($\phi$)

1) The diagonal BD is increased to BD'

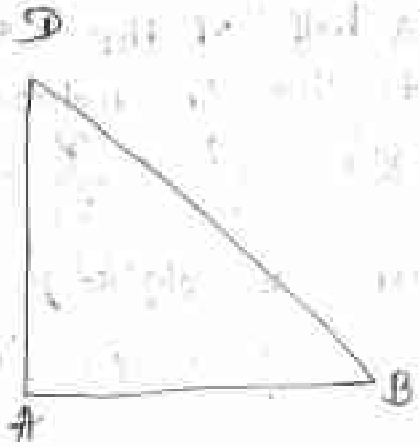
$$\text{Strain in BD} = \frac{\text{Change in Length}}{\text{original length}}$$

$$= \frac{\text{final length} - \text{Initial length}}{\text{Initial Length}}$$

$$= \frac{BD_1 - BD}{BD} \qquad BD = BD_2$$

$$= \frac{BD_1 - BD_2}{BD}$$

$$= \frac{D_1D_2}{BD}$$



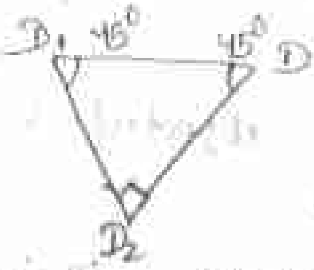

$$BD^2 = AD^2 + AB^2$$

$$BD^2 = AD^2 + AD^2$$

$$BD^2 = 2AD^2$$

$$BD^2 = \sqrt{2AD^2}$$

$$= \sqrt{2}AD \quad \text{—— (1)}$$



$$\cos 45^0 = \frac{D_1D_2}{DD_1}$$

$$\Rightarrow D_1D_2 = DD_1 \cos 45^0 \text{ —— (ii)}$$

$$\text{Strain } BD = \frac{DD_2}{BD}$$

Put the value of eq (i) and eq$^n$ (ii) in the above eq$^n$

$$\text{strain } BD = \frac{DD_1 \cos 45^0}{\sqrt{2}\, AD}$$

$$\text{strain in } BD = \frac{DD_1}{\sqrt{2}\times\sqrt{2}\, AD}$$

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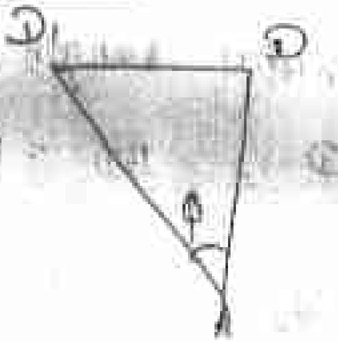
$$= \frac{DD_1}{2\,AD}$$



$$\tan\phi = \frac{DD_1}{AD}$$

$\phi$ is very small $\tan\phi = \phi$

$$\phi = \frac{DD_1}{AD}$$

$$\text{Strain in } BD = \frac{\phi}{2}$$

Thus the linear strain of the diagonal BD is half of the shear strain and is tensile in nature.

$$\text{linear strain } BD = \frac{\phi}{2} = \frac{\tau}{2C} \text{ —— (iii)}$$

[ Modulus of rigidity

$$C = \frac{\tau}{\phi} \Rightarrow \phi = \frac{\tau}{C}$$ ]

$\tau \rightarrow$ shear stress

$C \longrightarrow$ modulus of rigidity.

tensile strain on the diagonal BD due to tensile stress on the diagonal BD $= \frac{\tau}{E}$

tensile strain on the diagonal BD due to compressive stress on the diagonal AC $= \mu \frac{\tau}{E}$ —— (iv)

So Total strain in BD $= \frac{\tau}{E} + \mu \frac{\tau}{E}$

$$\Rightarrow \frac{\tau}{E}(1+\mu) \text{ —— (v)}$$

Compare eq$^n$ (iii) and (v)

$$\frac{\tau}{2C} = \frac{\tau}{E}(1+\mu)$$

$$\boxed{E = 2C(1+\mu)}$$ Formula

A, B, C, D, E, F, G, H, I, J, K

0, 1, 2, 3

$$1E(1-0\mu) = 2G(1+1\mu) = 3K(1-2\mu)$$

$$E = 2G(1+\mu) = 3K(1-2\mu)$$

prob - I

For a given material, young's modulus (E) = 120 Gpa, find bulk modulus and lateral contraction of a round bar of 50 mm in diameter and 2.5 m long when stretched 2.5 mm. Take poisson's ratio as 0.25

Soln: young's modulus (E) = 120 Gpa = $120 \times 10^3$ N/m²

Dia of bar (d) = 50 mm.

Length of bar (L) = 2.5 m = 2500 mm

Change in length ($\Delta L$) = 2.5 mm

$\mu$ = 0.25

$E = 3K(1-2\mu)$

$$K = \frac{E}{3(1-2\mu)}$$

Bulk modulus $K = \dfrac{E}{3(1-2\mu)}$

$$= \frac{120 \times 10^3}{3(1-2 \times 0.25)}$$

$= 80 \times 10^3$ N/mm²

= 80 Gpa

$$\mu = \frac{\text{longitudinal strain}}{\text{lateral strain}}$$

=> ~~Lateral~~ Longitudinal strain = $\mu \times$ lateral strain

$$\Rightarrow \text{Lateral strain} = \frac{\text{longitudinal strain}}{\mu}$$

Longitudinal strain $= \frac{\delta_L}{L} = \frac{2.5}{2.5\times10^3}$

$= \frac{1}{100} = 0.001$

Lateral strain $= \frac{0.001}{0.25} = 4\times10^{-3}$

$\frac{\Delta d}{d} = 4\times10^{-3}$

$\Rightarrow \Delta d = 4\times10^{-3}\times 50 = 0.2\,mm$

// Ans

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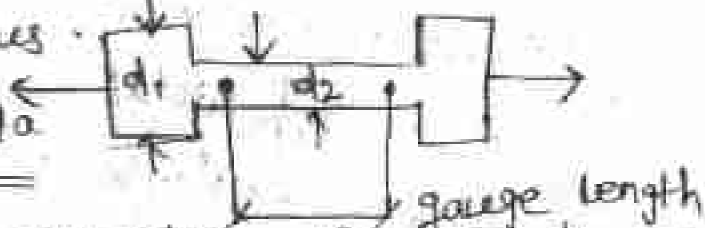
Prob - II In an experiment, a bar of 30mm dia is subjected to a pull of 60 kN. The measured extention on gauge length of 200mm is 0.09 m.m and the change in diameter is 0.0039 mm. Calculate the Poisson's ratio and the values of the three modules.




Given data

step - I Diameter of bar (d) = 30 m.m.

Tensile pull (P) = 60kN = $60\times10^3$ N

Length of specimen (L) = 200m.

Extention in length ($\Delta L$) = 0.09 m.m.

change in diameter ($\Delta L$) = 0.0039 m.m

step - II poisson's Ratio ($\mu$) = $\frac{\text{Longitudinal strain}}{\text{Lateral strain}}$

$= \frac{\frac{\Delta L}{L}}{\frac{\Delta d}{d}}$

Longitudinal strain $= \dfrac{\Delta l}{l}$

$$= \frac{0.09}{200}$$

$$= 0.00045$$

Lateral strain $= \dfrac{\Delta d}{d}$

$$= \frac{0.0039}{30}$$

$$= 0.00013.$$

$$\mu = \frac{\frac{\Delta l}{l}}{\frac{\Delta d}{d}} = \frac{0.00045}{0.00013}$$

$$= 3.461$$

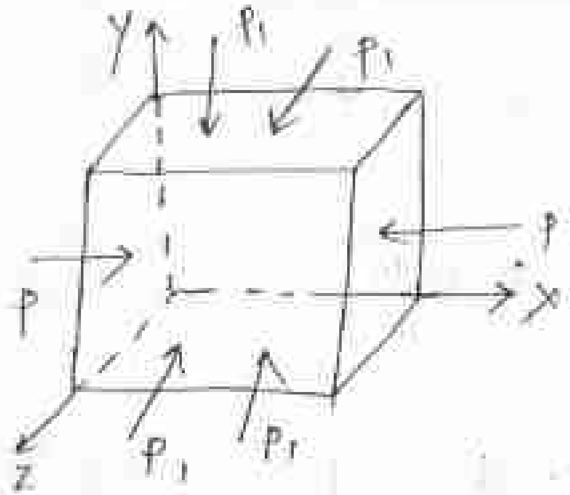
ie. k.t $\Delta l = \dfrac{PL}{AE} = 3.461$

$$\Delta\, 0.09 = \frac{60\times10^3\times200}{\frac{\pi}{4}\times30^2\times E}$$

Young's modulus $E = \dfrac{60\times10^3\times200}{\frac{\pi}{4}\times30^2\times0.09}$

$$= 188552.19$$

Problem 1-3 A cubical block is subjected to compression load 'p' in one direction if the lateral strain in other two direction are to be completely prevented by the application of another compressive load 'p'' find the value 'p'' in terms of p.




Step-I A cubical block ABCDEFGH and load on the opposite faces AEDH and BEGC = p (Compress). The other two faces will be subjected to lateral tensile strain. Now in order to present the lateral strains other two directions, we have to applied a compressive load of P1 lateral strain 'y' direction.

$$\frac{\sigma_y}{E}$$

$$\Rightarrow 0 = \left(\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}\right)$$

$$\Rightarrow 0 = \left(\frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E}\right)$$

$$\Rightarrow 0 = \frac{1}{E}\left(\sigma_y - \mu\sigma_x - \mu\sigma_z\right)$$

$$\Rightarrow 0 = \sigma_y - \mu\sigma_x = \mu\sigma_z$$

$$\Rightarrow 0 = \frac{P_1}{A} - \mu\frac{P}{A} - \mu\frac{P_1}{A}$$

$$\Rightarrow 0 = \frac{1}{A}\times\left(P_1 - \mu P - \mu P_1\right)$$

$$\Rightarrow 0 = \left(P_1 - \mu P - \mu P_1\right)$$

$$\Rightarrow 0 = (1-\mu)P_1 - \mu P$$

$$\Rightarrow \mu P = (1-\mu)P_1$$

$$P_1 = \frac{\frac{1}{\mu} P}{\left(1 - \frac{1}{\mu}\right)}$$

$$= \frac{\frac{P}{\mu}}{\frac{(\mu - 1)}{\mu}} = \frac{P}{m} \times \frac{m}{m-1}$$

$$\Rightarrow \boxed{P_1 = \frac{P}{m-1}}$$

Ans

Stress and strain curve for ductile and brittle material :-

Ducitle Material :-

The material which have ductility property that material is called as ductile materials

ductility The property by virtue which the material has large reduction in crossection and high degree of deformation under the tensile load is known as ductivity.

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It directly depends upon the % elongation

$$\% \text{ elongation} = \frac{\text{Final length} - \text{initial length}}{\text{Initial length}}$$

$\%E < 5\% \rightarrow$ Brittle material

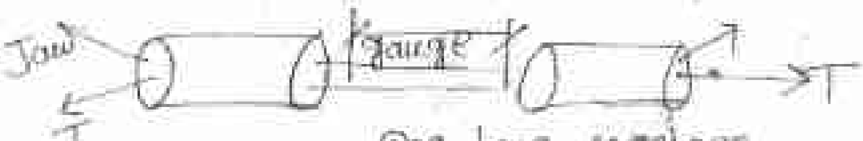
$5\% < E\% < 15\% \rightarrow$ Intermediate ductile material

E% > 15% → ductile material

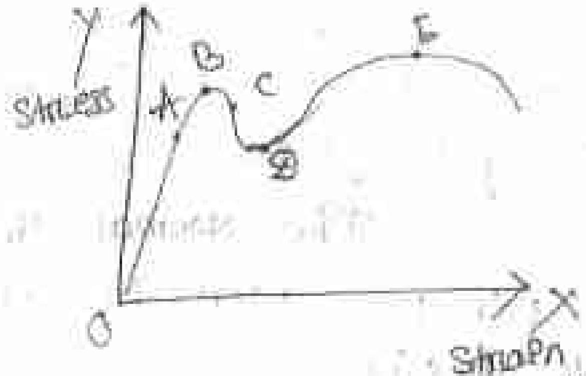
Stress - strain curve of ductile material:-

> This test is performed by the equipment it is called UTM → universal testing machine.

> If the carbon content % is less. That material is called as ductile material (Mild steel) (Fe250)




Dog bone specimen




OA → proportional limit
The limit of which the stress is directly proportional to the strain.

A to B → Elastic limit
The body returns back to its original position after removal of external force. That limit is called as elastic limit

B to C → upper yelding
The point at which the material strain to yelding.

C - D → lower yeild point :-

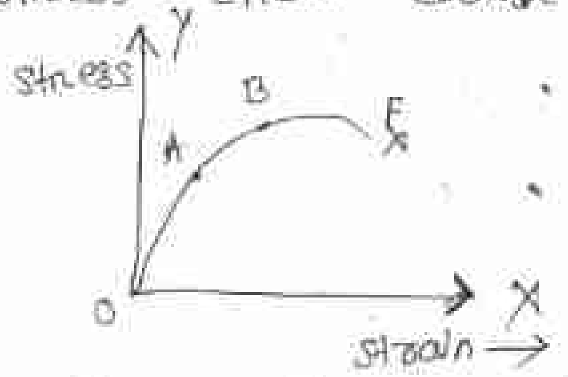
strain is more 'D' than point C.

D - E → ultimate point.

The point at which the material is achived it's max^m stress of failure is called as ultimate point.

D - F → breaking point.

stress - strain course of brittle material :-



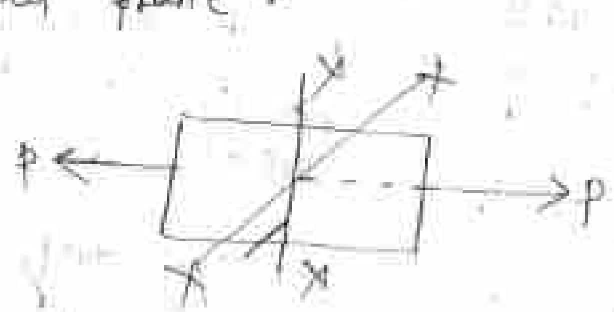

This is the curve of brittle material.

New chapter

## Principal stress & strains (UNIT-4)

**Principal stress** :- The normal stress acting on a principal plane is called as principal stress.

principal plane :- The plane which have no shear stress is called as principal plane.



$$\text{Stress} = \frac{\text{Resisting force}}{\text{Area}}$$

Method of determining stress on oblique (Inclined) plane :-

> There are two methods for determining the stress on oblique plane.

(i) Analytical Method.

(ii) Graphical method.

(i) Analytical method for determining stress on oblique plane :-

- The following three cases will be considered.

(i) A member is subjected to axial (or) direct stress in one plane.

~~(i) A member~~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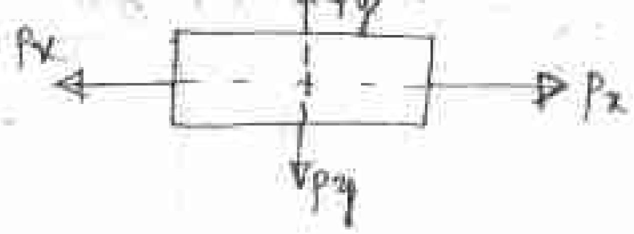
(ii) A member is subjected to stress in two mutually perpendicular direction.




(iii) A member subjected to shear stress

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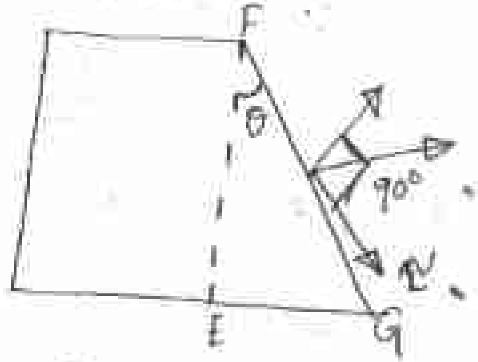
$$= \frac{P\sin\theta}{\frac{A}{\cos\theta}} = 2\frac{P}{A}\times\frac{1}{2}\sin\theta\cdot\cos\theta$$

$$= \frac{\sigma}{2}\sin 2\theta$$

Resultant stress :-




$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$\sigma_R^2 = \sigma_N^2 + \tau^2 + 2\sigma_N\tau\cos 90^\circ$$

$$\sigma_R^2 = \sigma_N^2 + \tau^2$$

$$\sigma_R = \sqrt{\sigma_N^2 + \tau^2}$$

The normal stress is max^m in an inclined plane when $\cos^2\theta$ or $\cos\theta$ is max^m

$$\sigma_N = \sigma\cos^2\theta$$

when $\theta = 0^\circ$ $\cos^2\theta = 1$

$$(\sigma_N)\ max^m = \sigma$$

That means the line FG coincide with the line FE

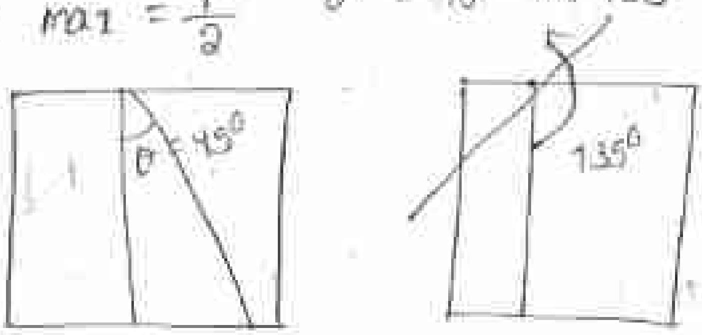
Max^m tangential stress in an inclined plan when it is subjected to direct stress in one direction if $\sin 2\theta$ value is max$^m$ $\tau = \frac{\sigma}{2}\sin 2\theta$

$\sin 2\theta = 1$

$2\theta = 90^0$ or $270^0$

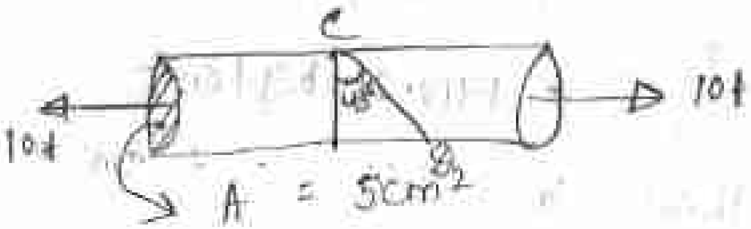
$\theta = 45^0$ or $135^0$

$\tau_{max} = \frac{\sigma}{2}$ $\quad \theta = 45^0$ or $135^0$




Problem - 1

A cast iron block of $5\,cm^2$ is subjected to a pull of 10t in the one direction. find out the resultant stress a plane which is inclined an angle of $45^0$ with the vertical.

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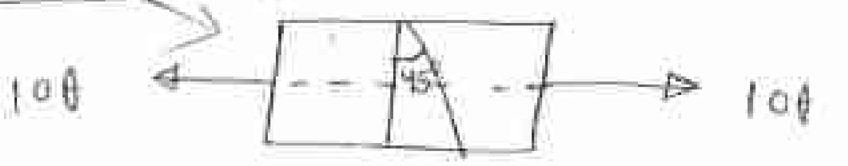
Sol$^n$


2D Draw



Step - 1

Data given :-

Area of cross section of cast iron block $(A) = 5 cm^2$.

Axial pull $(P) = 10 t$

$= 10,000 kg$

Angle of oblique plane $(\theta) = 45^\circ$

Step - 2 Resultant stress at an angle of $45^\circ$, with vertical plane.

$$\sigma_R = \sqrt{\sigma_N^2 + \tau^2}$$

$$\sigma_N = \sigma \cos^2\theta \quad ; \quad \left[ \sigma = \frac{P}{A} = \frac{10,000}{5} = 2000 \ kg/cm^2 \right.$$

$$= 2000 \times (\cos^2 45^\circ)$$

$$= 1414.21 \ kg/cm^2$$

Shear stress $(\tau) = \frac{\sigma}{2} \sin 2\theta$

$$= \frac{2000}{2} \sin(2 \times 45^\circ)$$

$$= 1000 \times \sin 90^\circ$$

$$= 1000 \ kg/cm^2$$

Resultant stress $\sigma_R = \sqrt{\sigma_x^2 + \tau^2}$

$$= \sqrt{(1414.21)^2 + (1000)^2}$$

Case - II  A body is subjected to two mutual perpendicular direct stress -

en

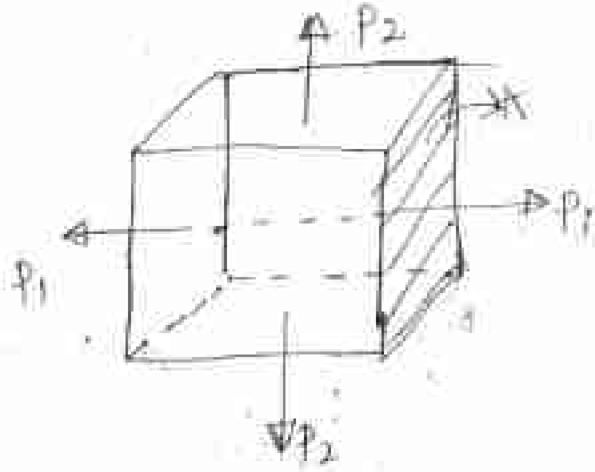
5°

angle

·

300

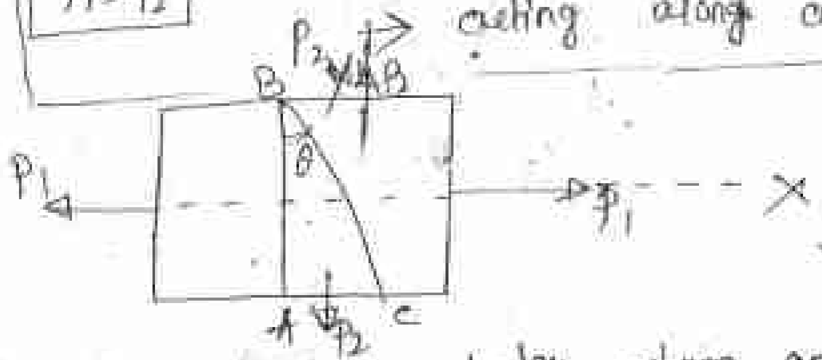
g/cm³



Let $P_1 \rightarrow$ acting along axis of major stress.

$\boxed{P_1 > P_2}$

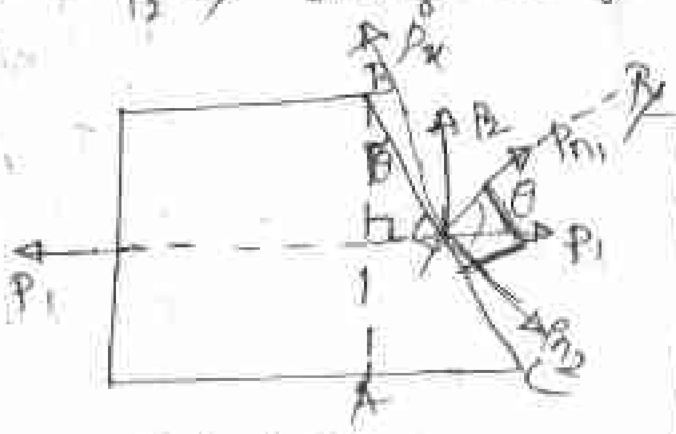
$P_2 \rightarrow$ acting along axis of minor stress.



Consider a body whose area of cross-section is 'A' Let it is subjected to two mutual perpendicular forces $P_1$ and $P_2$.

Let $P_1 \rightarrow$ acting along axis of major stress.

$P_2 \rightarrow$ acting along axis of minor stress.



$m\angle B = \theta$

$m\angle X = 90^\circ$

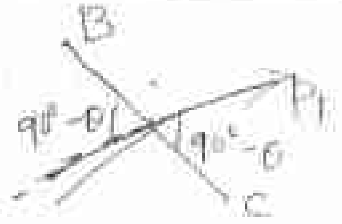
$m\angle Y = 180^\circ - (90^\circ + \theta)$

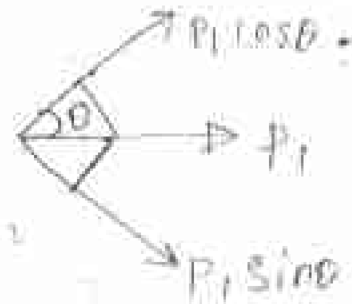
$= 180^\circ - 90^\circ - \theta$

$90^\circ - \theta$

$90^\circ - (90^\circ - \theta)$

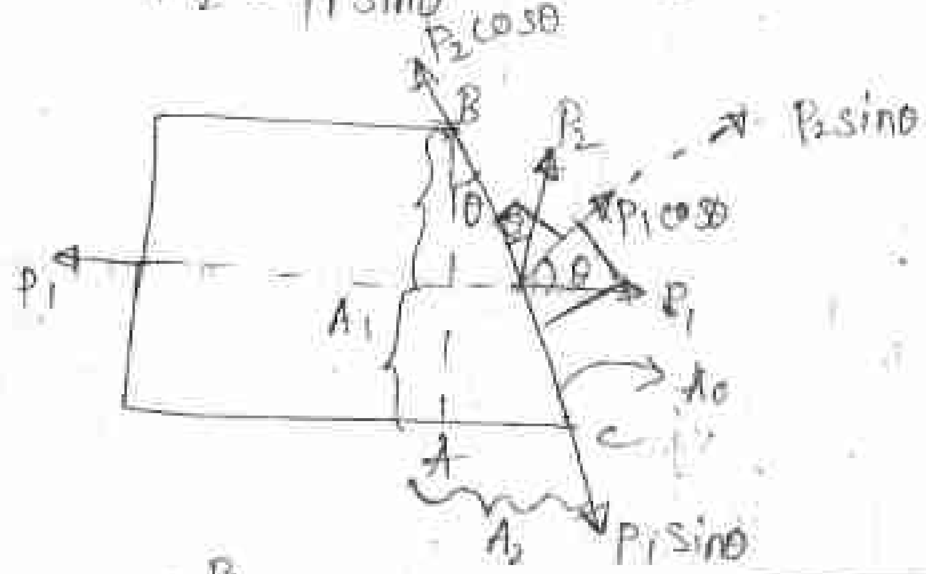
$= 90^\circ - 90^\circ + \theta$

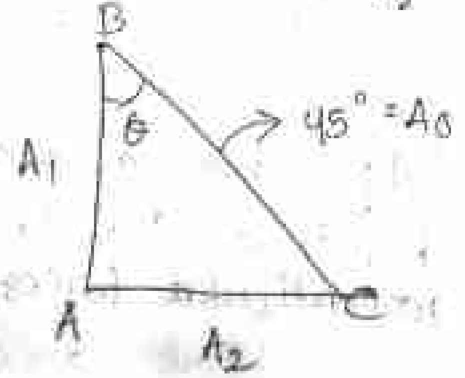



$P_{n_1} = P_1\cos\theta$

$P_{n_2} = P_1\sin\theta$




$$\cos\theta = \frac{A_1}{A_0} \quad \text{—(i)}$$

$$\sin\theta = \frac{A_2}{A_0} \quad \text{—(ii)}$$

$$A_1 = A_0\cos\theta$$

$$A_2 = A_0\sin\theta$$

$A_1 \rightarrow$ equivalent area of $A_0$ along '$x$' axis.

$A_2 \rightarrow$ equivalent area of $A_0$ along '$y$' axis.

Normal stress acting on an Inclined plane BC $= (\sigma_N) = \dfrac{\text{Normal Load}}{\text{Inclined Area}}$

$$= \frac{P_n}{A_0} \cdot \frac{P\cos\theta}{\frac{A}{\cos\theta}}$$

$= \frac{P}{A} \cos^2\theta = \sigma\cos^2\theta$

$$\boxed{\sigma_N = \sigma\cos^2\theta}$$

Tangential stress $(\sigma_T) = \tau$

$$= \frac{\text{Shear load}}{\text{Inclined area}} = \frac{P_T}{A_\theta}$$

$$= \frac{P_n \sin\theta}{\frac{A}{\cos\theta}}$$

$$= \frac{P}{A} \sin\theta \cdot \cos\theta = \tau\sin\theta \cdot \cos\theta$$

$$= \frac{2\tau}{2} \sin\theta \cdot \cos\theta$$

The normal stress in the inclined plane will be $max^m$ when $\cos^2\theta$ or $\cos\theta$ is $max^m$ $\cos\theta = max^m$ when

$\theta = 0^\circ$

$\cos 0^\circ = 1$.

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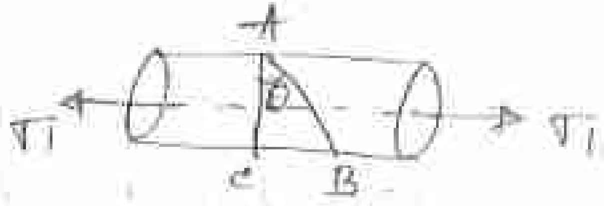
Formula

When a body is subjected to direct stress in one direction.

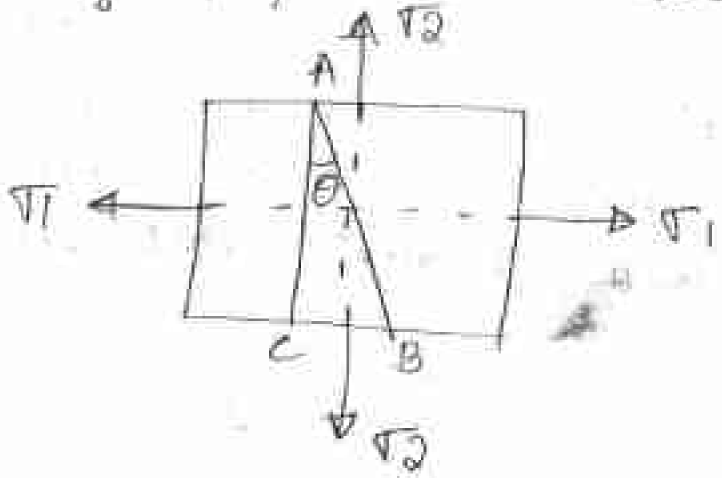



$$\sigma_N = \sigma_1 \cos^2\theta$$

$$\tau = \frac{\sigma_1}{2}\sin 2\theta$$

$$R = \sqrt{\sigma_N^2 + \tau^2}$$

When a body is subjected to two mutually perpendicular stresses.




$$\sigma_N = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$

$$\tau = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta$$

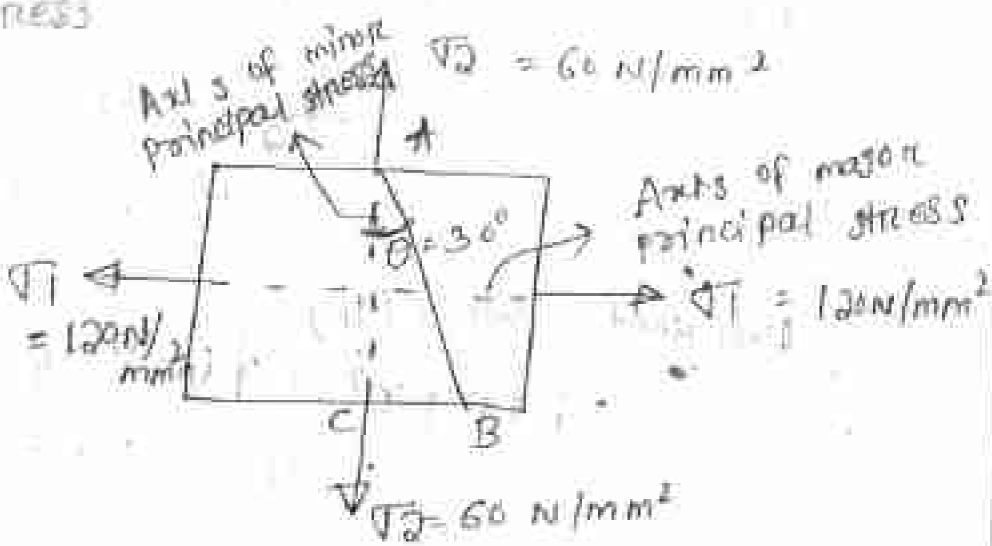
$$\sigma_R = \sqrt{\sigma_N^2 + \tau^2}$$

$$\phi = \tan^{-1}\left(\frac{\tau}{\sigma_N}\right)$$

$$\tau_{max} \frac{\sigma_1 - \sigma_2}{2}$$

Q1 The tensile stress at a point across two mutually perpendicular planes are 120 N/mm² (Tensile) and 60 N/mm² (tensile). Determine the normal, tangential and resultant stress on a plane at 30° to the axis of the minor stress.

Solⁿ




Step-1

Data given:-

Major principal stress $\sigma_1 = 120\ N/mm^2$ (Tensile)

Minor principal stress $\sigma_2 = 60\ N/mm^2$

$\theta \rightarrow$ angle which the plane makes with axis of minor principal stress $\theta = 30°$

Step-II

Normal stress $\sigma_N = \frac{\sigma_1 + \sigma_2}{2} + \left(\frac{\sigma_1 - \sigma_2}{2}\right)\cos 2\theta$

$= \frac{120 + 60}{2} + \left(\frac{120 - 60}{2}\right)\cos 60°$

$= 90 + 30 \cos 60^\circ$

$= 105$

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Tangential stress ($\tau$) $= \frac{\sigma_1 + \sigma_2}{2} \times \sin 2\theta$

$= \frac{120 - 60}{2} \sin(2 \times 30^\circ)$

$= 30 \sin 60^\circ = 25.98\ N/mm^2$.

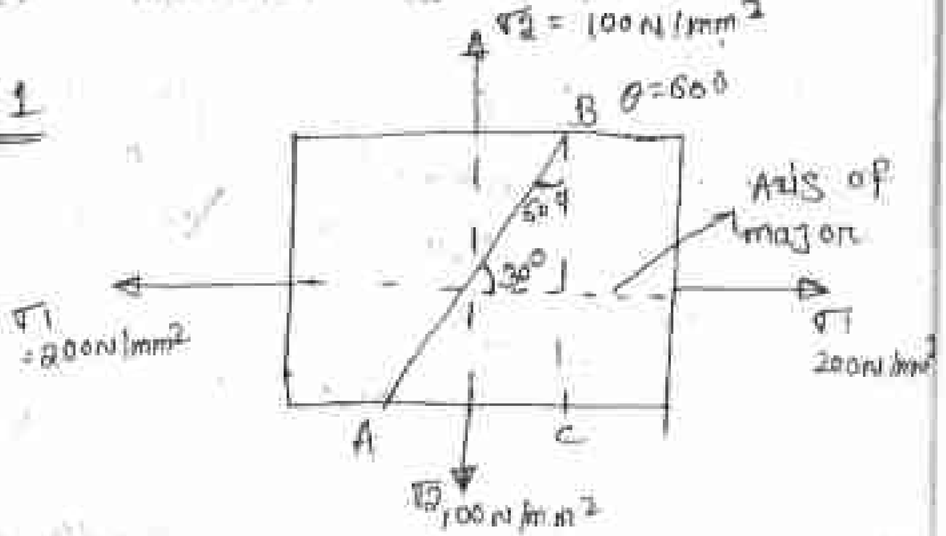
Resultant stress ($\sigma_R$)

$= \sqrt{\sigma_n^2 + \tau^2} = \sqrt{(105)^2 + (25.98)^2}$

$= 108.16\ N/mm^2$.

(Q.3) The stress [illegible] in a bar [illegible] 200 N/mm² (tensile) and [illegible] N/mm² (compressive). Determine the resultant stress in magnitude and direction on a plane [illegible] the axis of the major stress. Also determine the maximum intensity of shear stress in the material at the point.

Sol: Step-1



Step - II Data given :-

major principal stress $\sigma_1 = 200 N/mm^2$

Minor principal stress $\sigma_2 = -100 N/mm^2$

$\theta = 180^0 - (90^0 + 30^0)$

$= 60^0$

Step - III Normal stress $\sigma_N = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$

$= \frac{200 - 100}{2} + \frac{200 - 100}{2} \cos 120^0$

$= 50 + 150 \cos 120^0$

$= -25 N/mm^2$

Tangential stress $(\tau) = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta$

$= \frac{200 - (-100)}{2} \sin 120^0$

$= 129.90 N/mm^2$

$\sigma_R = \sqrt{(-25)^2 + (129.90)^2}$

$= 132.28 \ N/mm^2$

$\phi = \tan^{-1}\left(\frac{\tau}{\sigma_N}\right)$

$= \tan^{-1}\left(\frac{25}{129.90}\right)$

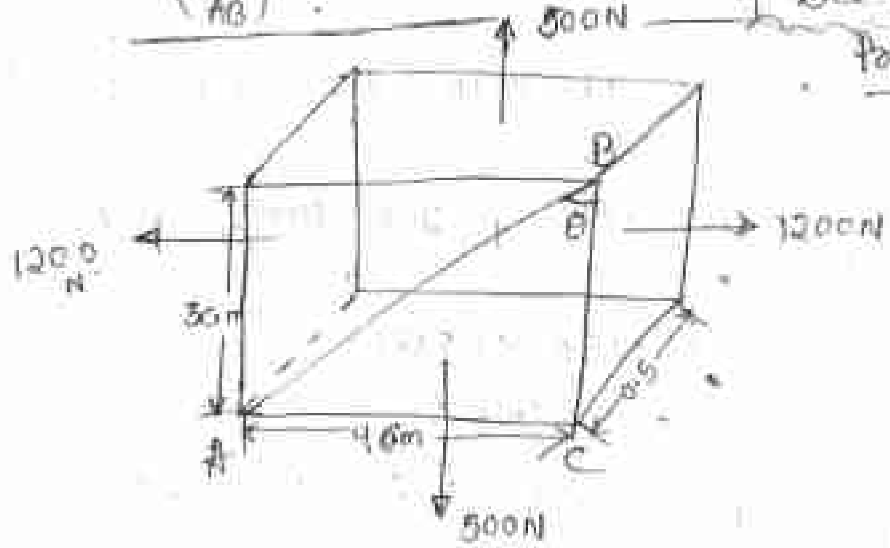
$= 10.89$

$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$

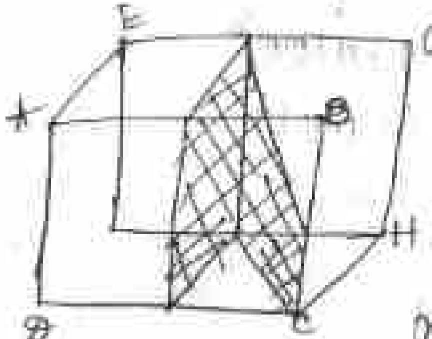
$= \frac{200 - (-100)}{2} = 150 N/mm^2$

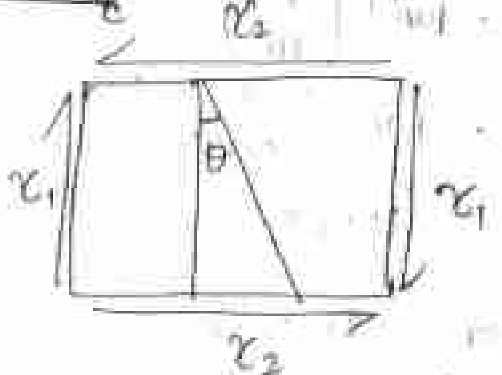
30 A small block is 4cm long, 3cm high and 0.5cm thick. It is subjected to uniformly distributed tensile forces of the resultants 1200N & 500N as shown in the figure. Calculate normal stress and shear stress develop on the diagonal AB.
9 Dec 2020

Prove Question



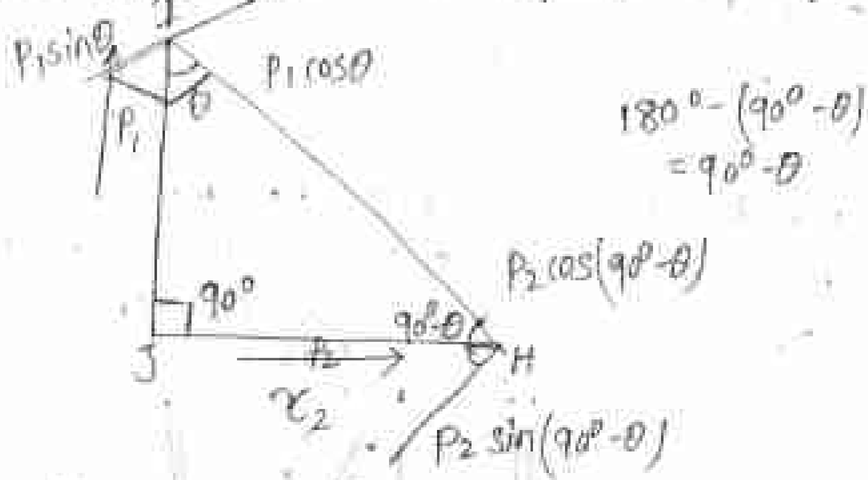

A body is subjected to shear stress




Let us consider a body whose crossectional area is 'A'. Let it is subjected to a positive shear stress along x-x axis. Now let us consider an oblique section EH inclined with y-y axis on which we are required to find

Out the stress in figure.




$\tau_1$ = positive shear stress along x-x axis

$\theta$ = Angle on which the oblique section

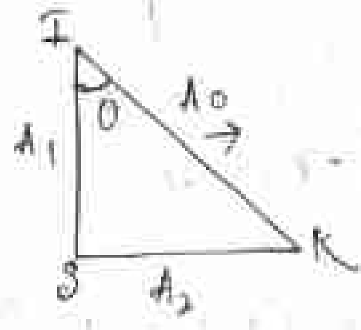
IH = Makes with y-y axis

Consider the equilibrium of the wedge ABC, As per principle of simple shear stress. The value of

$$\tau_1 = \tau_2, \; \tau_1 = \tau_2 = \tau$$

So the vertical force acting on IH be

$$P_n = P_1 \sin\theta + P_2 \sin(90^\circ - \theta)$$
$$= P_1 \sin\theta + P_2 \cos\theta$$




$$\cos\theta = \frac{A_1}{A_0}$$

$$A_0 = \frac{A_1}{\cos\theta}$$

$A_1$ = equivalent area of $A_1$ along x direction.

$$\sin\theta = \frac{A_2}{A_0}$$

$$A_0 = \frac{A_2}{\sin\theta}$$

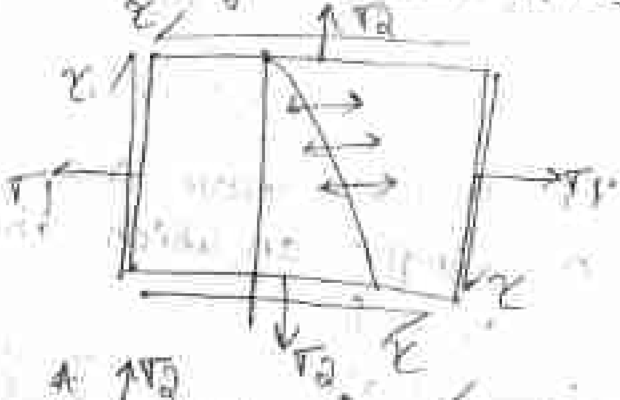
$A_2 \rightarrow$ equivalent area of $A_0$ along direction.

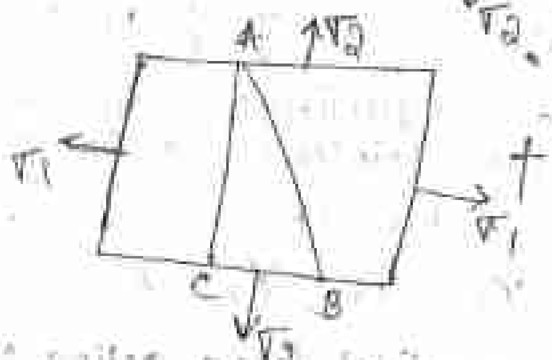
Normal stress across the section (IH)

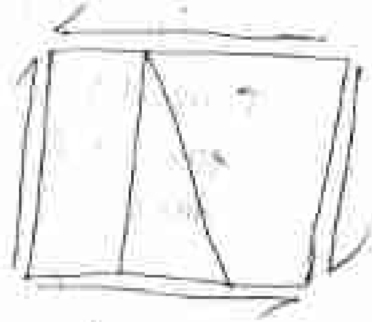
$$\sigma_n = \frac{P_n}{A_0} = \frac{P_1 \sin\theta}{A_0} + \frac{P_2 \cos\theta}{A_0}$$

$$= \frac{\frac{P_1 \sin\theta}{\frac{A_1}{\cos\theta}}}{} + \frac{P_2 \cos\theta}{\frac{A_2}{\sin\theta}}$$

A body is subjected to two mutually perpendicular direct stress and accomplined by shear stress.



$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$

$$\sigma_N = \frac{\sigma_1 - \sigma \sin\theta}{2}$$

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$\sigma_T = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - \tau\cos 2\theta$$

$$\sigma_n = \tau \sin 2\theta$$

$$\sigma_T = -\tau\cos 2\theta$$

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_T^2}$$

$$= \tau_1 \sin\theta \cos\theta + \tau_2 \sin\theta \cdot \cos\theta$$

$$\tau_1 = \tau_2 = \tau$$

$$= \tau \sin\theta \cdot \cos\theta + \tau \sin\theta \cdot \cos\theta$$

$$= 2\tau \sin\theta \cdot \cos\theta = \tau \sin 2\theta$$

Tangential force acting on inclined plane ($\sigma_N$)

$P_T = P_2 \cos(90^\circ - \theta) - P_1 \cos\theta$

$= P_2 \sin\theta - P_1 \cos\theta$

shear stress acting on the inclined plane

(I K) $(\tau \text{ or } \sigma_T) = \frac{P_2 \sin\theta}{A_0} - \frac{P_1 \cos\theta}{A_0}$

$(\tau \text{ or } \sigma_T) = \frac{P_2 \sin\theta}{\frac{A_2}{\sin\theta}} - \frac{P_1 \cos\theta}{\frac{A_1}{\cos\theta}}$

$= \tau_2 \sin^2\theta - \tau_1 \cos^2\theta$

$\because \tau_2 = \tau_1 = \tau$

$= \tau \sin^2\theta - \tau \cos^2\theta$

$= -\tau(\cos^2\theta - \sin^2\theta)$

11 Dec 2020 $= -\tau \cos 2\theta$ (Formula)

Position of Principal Plane :-

Principal plane - The plane which have no shear stress. That plane is known as principal plane. The stresses in the principal plane is known as principal stress.

We known that $\tau = 0$

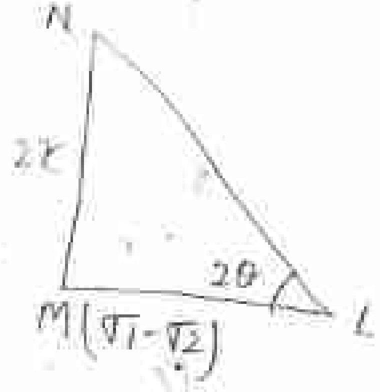
$\frac{\sigma_1 - \sigma_2}{2} \sin 2\theta - \tau \cos 2\theta = 0$

$\Rightarrow \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta = \tau \cos 2\theta$

$\Rightarrow \frac{\sin 2\theta}{\cos 2\theta} = \frac{\tau}{\frac{\sigma_1 - \sigma_2}{2}}$

$\Rightarrow \tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2}$

Now diagonal of the right angle tringle




$$= \pm\sqrt{(2\tau)^2 + (\sigma_1-\sigma_2)^2}$$

$$= \sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

OR $-\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}$

Case - I

diagonal $\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}$

$$\sin 2\theta = \frac{\text{Height}}{\text{Diagonal}} = \frac{2\tau}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}$$

$$\cos 2\theta = \frac{\text{base}}{\text{Diagonal}}$$

$$= \frac{(\sigma_1-\sigma_2)}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}$$

Major Principal stress

$$\sigma_N = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\cos 2\theta + \tau\sin 2\theta$$

$$= \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\left(\frac{\sigma_1-\sigma_2}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}\right) + \tau\left(\frac{2\tau}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}\right)$$

$$= \frac{\sigma_1+\sigma_2}{2} + \left[\frac{(\sigma_1-\sigma_2)(\sigma_1-\sigma_2)+2\tau\, 2\tau}{2\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}\right]$$

$$= \frac{\sigma_1+\sigma_2}{2} + \left[\frac{(\sigma_1-\sigma_2)^2+4\tau^2}{2\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}\right]$$

$$= \frac{\sigma_1-\sigma_2}{2} + \frac{1}{2}\left[\frac{(\sigma_1-\sigma_2)^2+4\tau^2}{\sqrt{(\sigma_1-\sigma_2)^2+4\tau^2}}\right]$$

$$= \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

Minor principal stress :-

$$\text{Diagonal} = \cancel{\phi} \sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

$$\sigma_N = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$= \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

15 Dec 2020

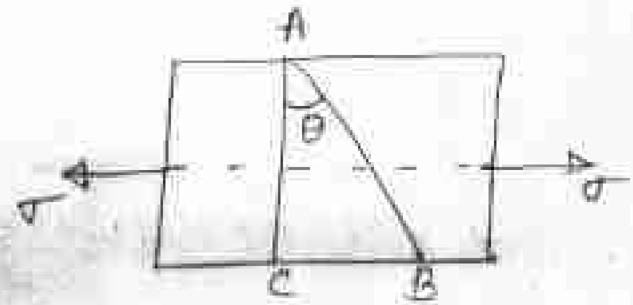
Graphical method (otto mohr)

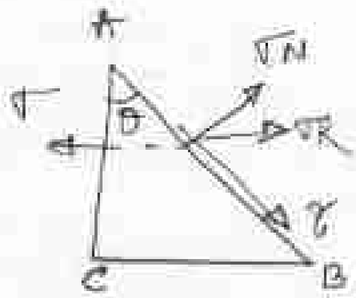
Mohr's circle method :-

~~Prob-1~~ ~~A Cast~~

Case - 1

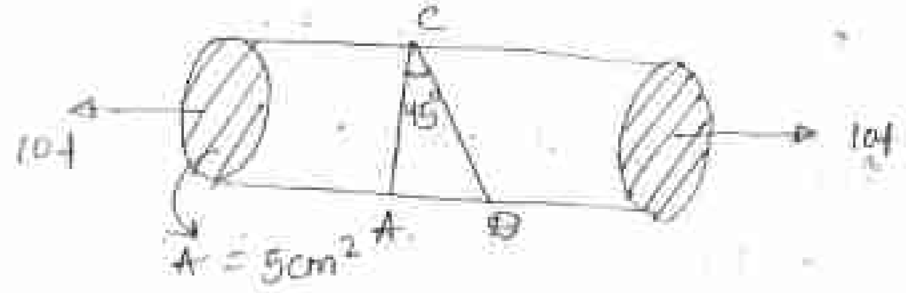
→ A body is subjected to direct stress in one direction.



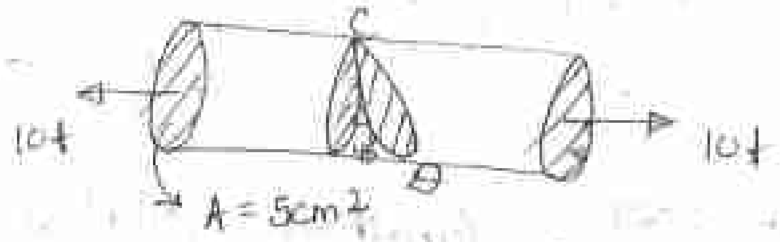




Step - 1 Construct coordinate system and normal stress is taken along x-axis and shear stress taken in y-axis.

Problem - 1 A cast iron block of $5cm^2$ is subjected to a pull of 10 t in one direction. Find out the resultant stress a plane which is inclined at an angle of 45° with the vertical.




Setep-1




Force acting on the cast iron block

$= \frac{Force}{Area} = \frac{10t}{5} = 2t$

Step-2

Shear stress AB = 1 cm = 1 t / cm²

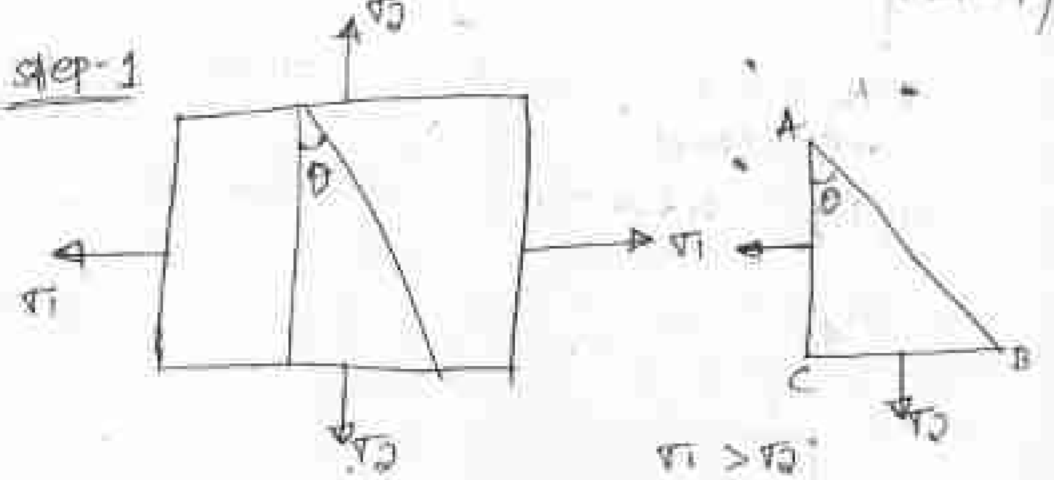
OB = 1.5 cm

$(\sigma_R) = 1.5 \, t/cm^2$

$[\sigma_N]$ OA = 1 cm = 1 t / cm²

Case - II    A body is subjected to two mutual perpendicular stress.
(Mohr's circle method)

step - 1




$\sigma_1 > \sigma_2$

step - 2

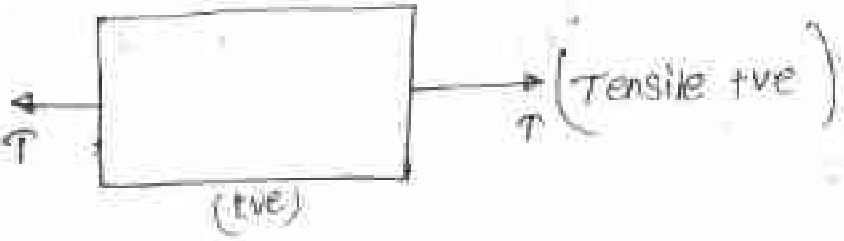
16 Dec 2020

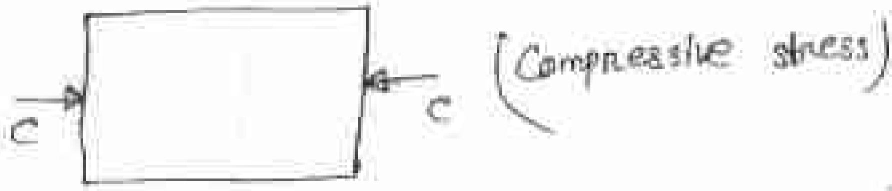
Graphical method to determine the complex stress in an Inclined plane :-

Normal Stress
Tangential stress
Resultant stress

→ It is a geometrical method to determinate the complex stress.
→ By ottomohr's we have to

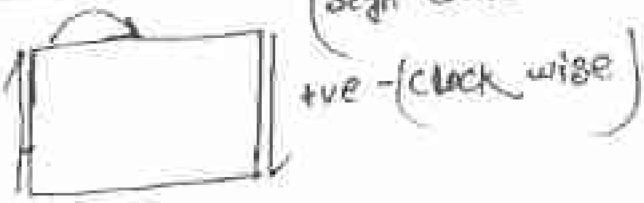
Sign convention for mohr's circle method

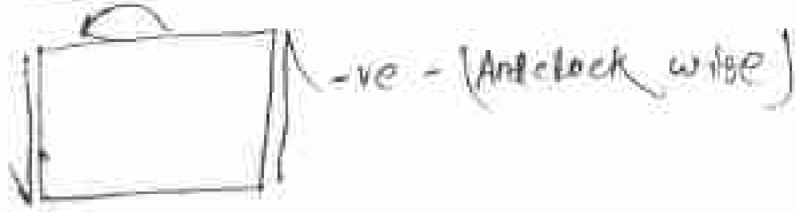


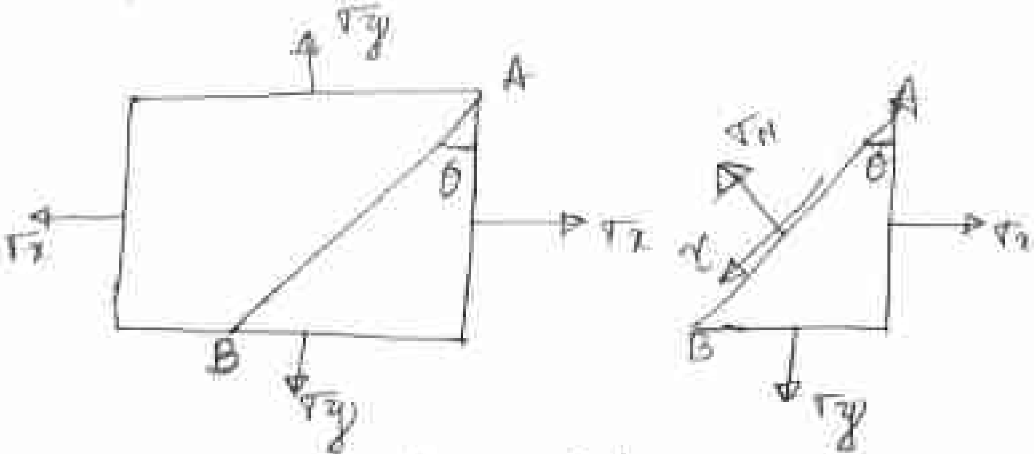

Tangential or shear stress :-

(Sign convention for mohr's circle method)



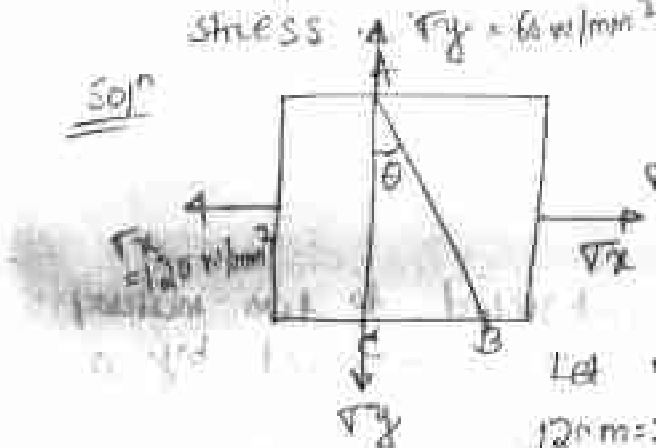
⇒ A body is subjected to two mutual perpendicular stress :-




$(\sigma_x > \sigma_y)$

Q) The tensile stress at a point across two mutually perpendicular planes are 120N (Tensile) & 60 N/m² (Tensile) determine the tangential and resultant stress on a plane at 30° the axis of the minor stress

Sol$^n$




$\theta = 30^0$

$\sigma_y = 60\,N/mm^2$

1cm = 10 N/mm²

60 N/mm²

Let $\sigma_x = 120\,N/mm^2$

12cm = 120 N/mm²

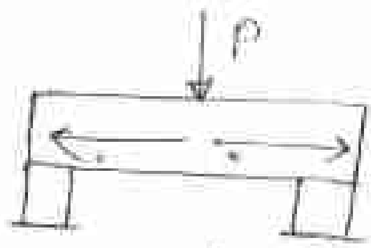
1cm = 10 N/mm²

23 Oct 2020

# Bending of beams

## Beam :-

Beams are structural members which are used to transfer lateral load's / vertical load.

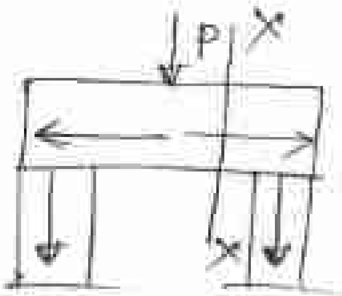


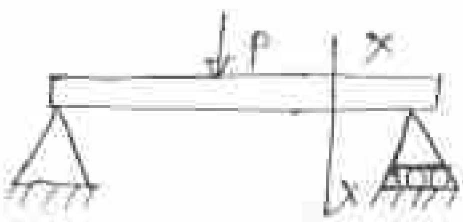

> Suppose consider a beam carrying a point load at its Centre.

> The vertical load is transfered to the horizontal beam and finally transfer to the Column.

> By the application of this load The B.M and S.F is developed in the beam.

> If we ~~can~~ cut a section and draw the free body diagram as shown in the fig.




F·B·D

* A shear force and bending moment will devlop due to this shear force. Shear stress will devlop.

$$\boxed{\tau = \frac{FQ}{Ib}}$$

* Due to bending moment bending stress will be developed.

$$\boxed{\sigma_b = \frac{M}{Z}}$$

Bending stress :-

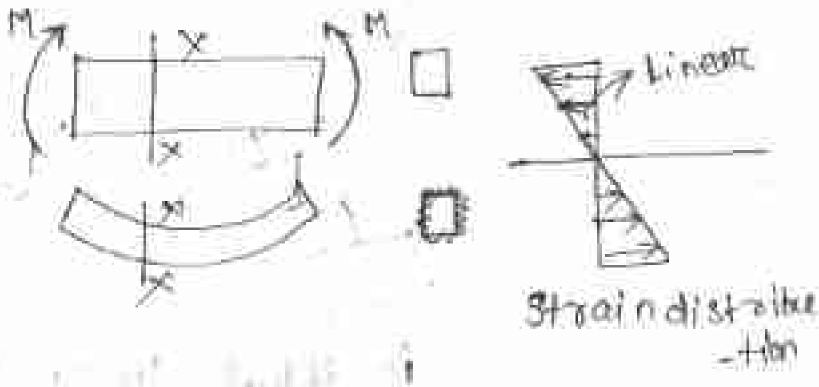
The resistance offered by the internal stresses to bending is called as bending stress.

Exam-Q

Assumption for the theory of simple bending

xox ———— xox

1. Plane sections remains plane even after bending. As per this assumption, there is no warping and twisting in the cross-section of the beam

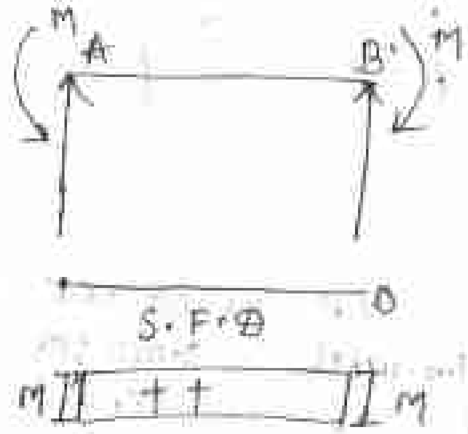


It implies that the strain distribution will be linear up to failure.

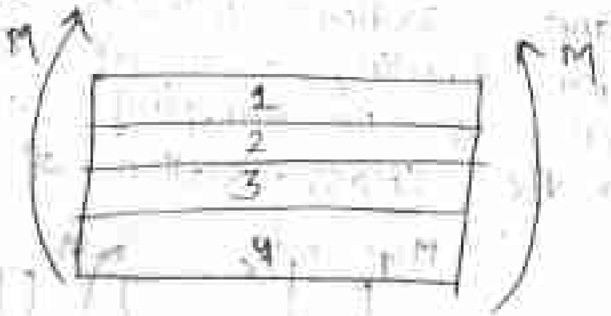
D.M.T.
(2) The beam material is homogenious isotropic and follows Hook's law.

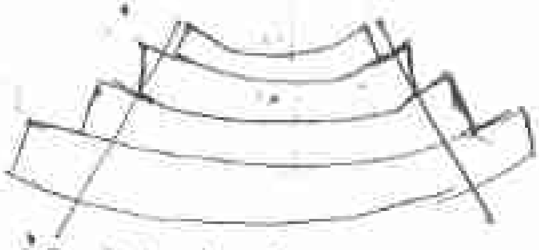
Isotropic - The values of 'E' in all direction will be same.
It means that the formula derived in the eqn are within elastic limit.

(3) Beams are subjected to pure bending i.e. there is no shear force in the beam section.



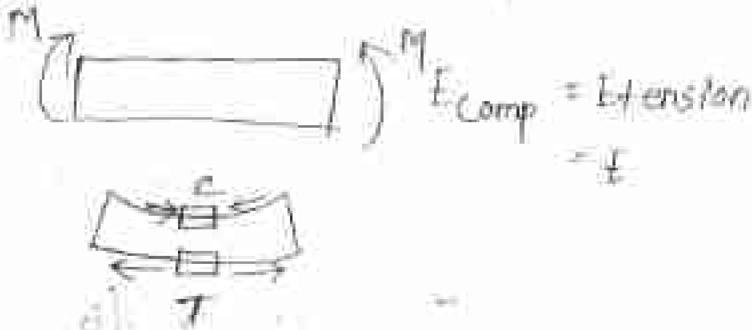

(4) Each layer is free to expand and contract.



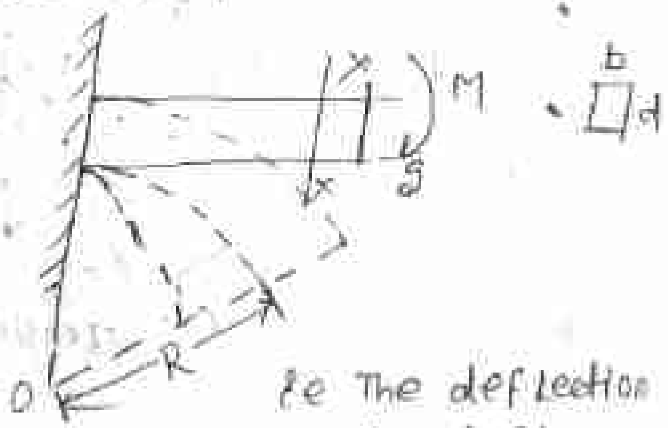
i.e. it bends in an arc of a circle.

(5) The values of E all materials are Same.




(6) The radious of curvature is greater than width and depth of beam.

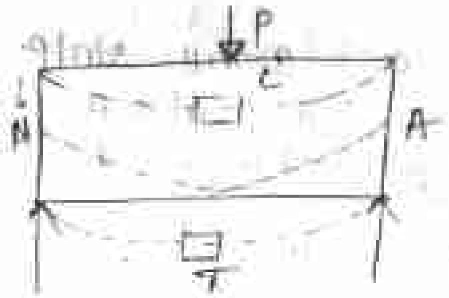
$R >> b, d$




ie the deflection will very less.

So the formula can be used for less deflection.

Theory of simple bending :-




Consider a beam and let it is subjected to a load. so that the top layer is subjected to Compression and bottom layer is subjected to tension.
But there is a layer where neither tension nor compression stress is subjected.

$$\boxed{\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}}$$

M = moment of resistance
I = Moment of Inertia
$\sigma_b$ = bending stress
y → distance of N·A from extreme fibre
E → young's modulus
R → Radious of Curvature

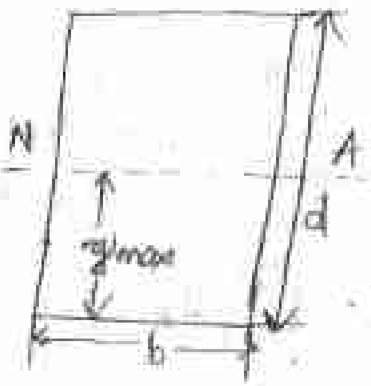
Section modulus (Z) :-

It is the ratio of moment of inertia to max^m distance of extreme fibre from N-A

$$Z = \frac{I}{y_{max}}$$

Section modulus for different sections

① Rectangular



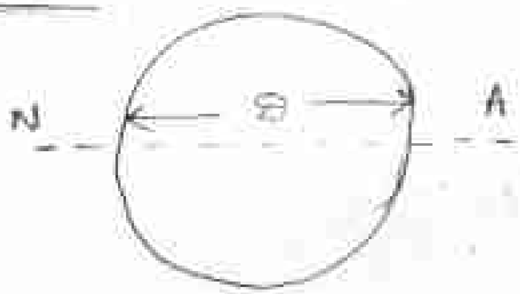

$$Z = \frac{I}{y_{max}}$$

$$I = \frac{bd^3}{12}$$

$$y_{max} = \frac{d}{2}$$

$$Z = \frac{\frac{bd^3}{12}}{\frac{d}{2}} = \frac{bd^2}{6}$$

* Circular




$$Z = \frac{I}{y_{max}}$$

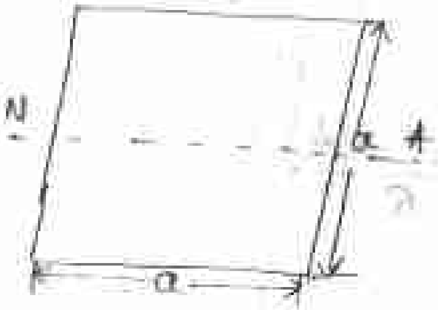
$$I = \frac{\pi}{64} D^4$$

$$y_{max} = \frac{D}{2}$$

$$Z = \frac{\frac{\pi}{64} \times D^4}{\frac{D}{2}}$$

$$= \frac{\pi}{32} D^3$$

*** Squre**




$$Z = \frac{I}{y_{max}}$$

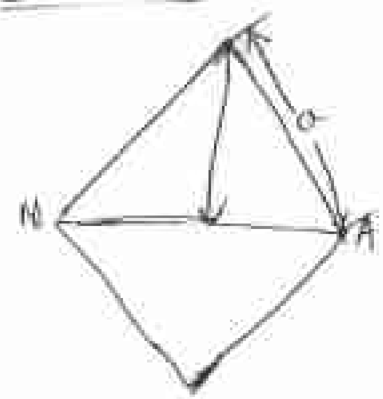
$$I = \frac{a \times a^3}{12}$$

$$= \frac{a^4}{12}$$

$$y_{max} = \frac{a}{2}$$

$$Z = \frac{\frac{a^4}{12}}{\frac{a}{2}} = \frac{a^3}{6}$$

*** Diamond**




$$Z = \frac{I}{y_{max}}$$

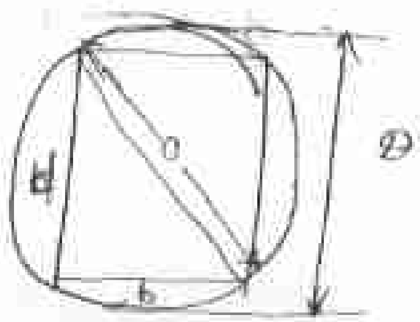
$$I = \frac{a^4}{12}$$

$$y_{max} = \frac{a}{\sqrt{2}}$$

$$Z = \frac{\frac{a^4}{12}}{\frac{a}{\sqrt{2}}} = \frac{a^3}{6\sqrt{2}}$$

**10** A rectangular beam is to be cut from circular log of wood of diameter 'D' find the ratio of dimensions for the strongest section in bending.

**Ans** The strength section will be $Z = \frac{bd^2}{6}$




From geometrical fig

$$D = \sqrt{b^2 + d^2}$$

$$\Rightarrow D^2 = b^2 + d^2$$

$$\Rightarrow d = \sqrt{D^2 - b^2}$$

$$\Rightarrow d^2 = D^2 - b^2$$

$$Z = \frac{bd^2}{6} = \frac{b(D^2 - b^2)}{6}$$

$$= \frac{bD^2 - b^3}{6}$$

$$\therefore \frac{dZ}{db} = 0$$

$$\Rightarrow \frac{d}{db}\left(\frac{bd^2 - b^3}{6}\right)$$

$$\Rightarrow \frac{1}{6}\left[\frac{d}{db}(bd^2) - \frac{d\,b^3}{db}\right]$$

$$\Rightarrow \frac{1}{6}\left[d^2 \cdot 1 - 3b^2\right]$$

$$\Rightarrow \frac{1}{6}\left[d^2 - 3b^2\right]$$

$$\Rightarrow \frac{1}{6}\left[D^2 - 3b^2\right] = 0$$

$$\Rightarrow D^2 - 3b^2 = 0$$

$$\Rightarrow D^2 = 3b^2$$

$$\Rightarrow b = \sqrt{\frac{D^2}{3}} = \frac{D}{\sqrt{3}}$$

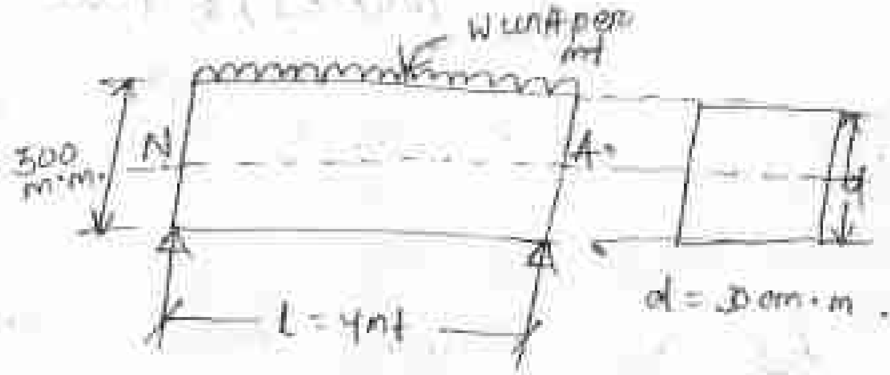
$$d^2 = D^2 - b^2$$

$$\Rightarrow \frac{d}{6}\left[\cancel{D^2 - 3b^2}\right] = 0 \quad d = \sqrt{\frac{2}{3}}\, D$$

28 Dec 2020

Q2 A rectangular beam 300m.m deep is simply supported over a span of 4mt what uniformly distributed load per mt the beam can carry if the bending stress is not exceed to 120N/mm² . Take $I = 8\times10^6\,mm^4$ .

Step-1




Step-2

depth of the beam (d) = 300m.m.

Moment of inertia (I) = $8\times10^6$ m.m.

span of the beam (l) = 4m = 4000m.

bending stress ($\sigma_b$) = 120 N/mm²

Step-3

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\Rightarrow M = \sigma_b \frac{I}{y} \qquad \left[ \frac{I}{y_{max}} = Z \right]$$

$$M = \sigma_b Z$$

$y$ = Distance of extreme fibre from N.A

$$y = \frac{D}{2} = \frac{300}{2} = 150\,m.m.$$

$$Z = \frac{I}{y} = \frac{8\times10^6\,mm^4}{150\,m.m.} = 53{,}334.33\,mm^3$$

Let $w$ be the weight on the simply supported beam.

$$M = \frac{Wl^2}{8} = \frac{w(4000)^2}{8} = 2000,000\,w$$

we know that $M = \sigma_b Z$

$\Rightarrow 2000,000\,w = 120 \times 53,334.33$

$$\Rightarrow w = \frac{120 \times 53,334.33}{2000.000}$$

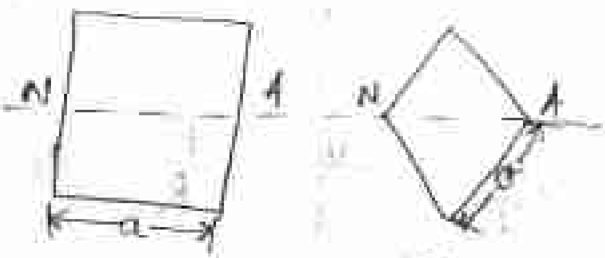
$w = 3200.0$

**Q3** For a given stress compare the moment of resistance of a beam of a square section when placed.

(a) with it's ~~diagona~~ sides horizontal.

(b) with it's diagonal horizontal.

Soln:-




(a) (Case-I) (b) (Case-II)

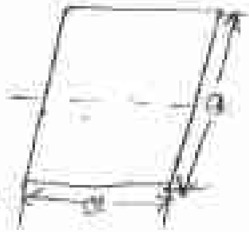
$M = \sigma_b Z$

**Case-I** $M_1 = \sigma_b Z_1$ $\left[\begin{array}{l} M_1 = \text{moment of resistance of section-I} \\ Z_1 = \text{section modulus of section-I} \end{array}\right.$

**Case-II** $M_2 = \sigma_b Z_2$ $\left[\begin{array}{l} M_2 = \text{moment of resistance of section-II} \\ Z_2 = \text{section of modulus of section-II} \end{array}\right.$

$$M_1 : M_2 = \frac{M_1}{M_2} = \frac{\sigma_b Z_1}{\sigma_b Z_2} = \frac{Z_1}{Z_2}$$

$Z_1$ = Section modulus of section-I



$$Z_1 = \frac{I_1}{(y_{max})_1}$$

$I_1$ = moment of inertia of section-I

$(y_{max})_1 \rightarrow$ distance of extreme fibre from N.A

$$I_1 = \frac{bd^3}{12} \quad \because [b = a, \ d = a]$$

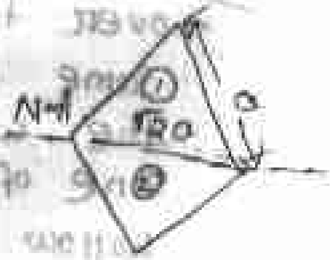
$$= \frac{a \times a^3}{12} = \frac{a^4}{12}$$

$$(y_{max})_1 = \frac{a}{2}$$

$$Z_1 = \frac{I_1}{(y_{max})_1} = \frac{\frac{a^4}{12}}{\frac{a}{2}} = \frac{a^4}{12} \times \frac{2}{a} = \frac{a^3}{6}$$

Case-II

$Z_2$ = section modulus of section = II



$$Z_2 = \frac{I_2}{(y_{max})_2}$$

$$I_2 = 2 \times \frac{bh^3}{12}$$

$$= 2 \times \frac{\sqrt{2}a \times \left(\frac{a}{\sqrt{2}}\right)^3}{12}$$

$$= 2 \times \frac{\sqrt{2}a}{12} \times \left(\frac{a}{\sqrt{2}}\right)^3 \qquad 2 = \sqrt{2} \times \sqrt{2}$$

$$(\sqrt{2})^3 = \sqrt{2} \times \sqrt{2} \times \sqrt{2} = 2\sqrt{2}$$

$$= 2 \times \frac{\sqrt{2}a}{12} \cdot \frac{a}{2\sqrt{2}}$$

$$= \frac{\sqrt{2}\, a^4}{12\sqrt{2}} = \frac{a^4}{12}$$

$$(y_{max})_2 = \frac{a}{\sqrt{2}}$$

$$Z_2 = \frac{\frac{a^4}{12}}{\frac{a}{\sqrt{2}}} = \frac{a^4}{12} \times \frac{\sqrt{2}}{a}$$

$$= \frac{a^3}{12} \times \sqrt{2}$$

$$= \frac{a^3}{6 \times 2} \times \sqrt{2} = \frac{a^3 \times \sqrt{2}}{6 \times \sqrt{2} \times \sqrt{2}}$$

$$= \frac{a^3}{6\sqrt{2}}$$

$$\frac{M_1}{M_2} = \frac{Z_1}{Z_2} = \frac{\frac{a^3}{6}}{\frac{a^3}{6\sqrt{2}}} = \frac{a^3}{6} \times \frac{6\sqrt{2}}{a^3}$$

$$\frac{M_1}{M_2} = \sqrt{2} \Rightarrow \boxed{M_1 = \sqrt{2}\, M_2}$$

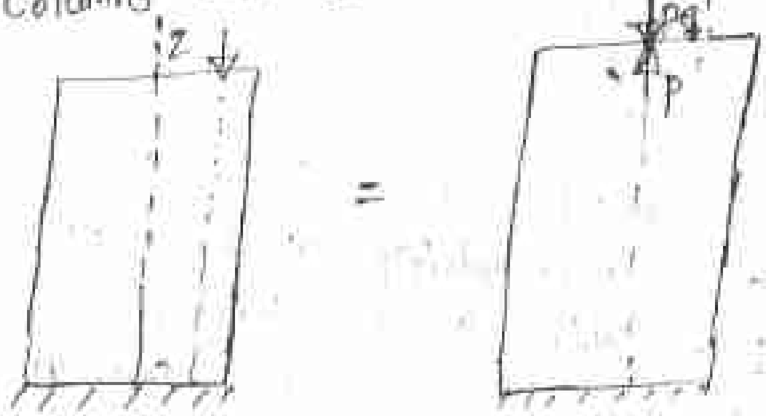
Q4 Two beams are simply supported over the same span and have the same flexural strength and compare the weight of these two beams. if one of them is solid and other is hollow circular with internal diameter is half of the external diameter.

4 January 2021

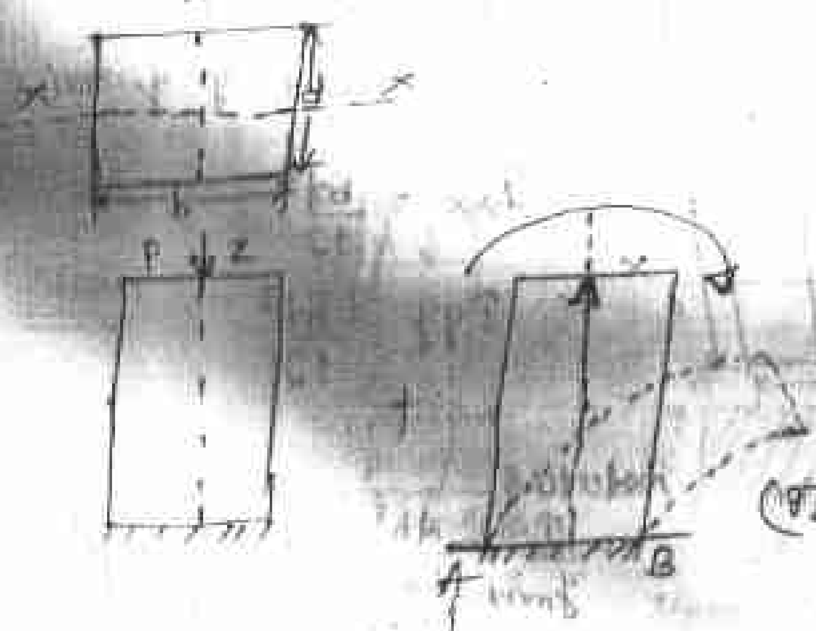
Direct and bending stress -

Column :- It is a vertical member fixed at both ends and subjected to compressible load.

Columns with eccentric Loading :-



elevation



$(\sigma_b max) = a + B$

Direct stress

$\sigma_0$

Bending stress

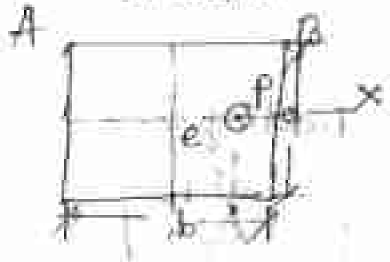
$\sigma_b$

$\sigma_b min$

Symmetrical columns with eccentric loading about one axis :-


Elevation

Consider a column ABCD subjected to an eccentric loading about one axis (i.e about y-y axis) as shown in the fig.




Plan

Let P = load acting on the column

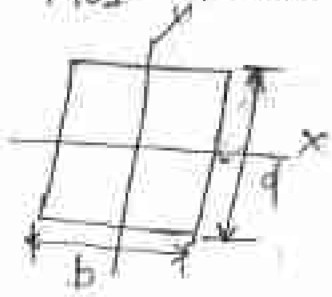
e = eccentricity of the load

b = width of the section

d = Thickness of the column

Area of the section $A = bd$

MOI of the section about y-y axis.




$$I_{xx} = \frac{bd^3}{12}$$

$$I_{yy} = \frac{bd^3}{12}$$

Section modulus

$$Z = \frac{I}{y_{max}} = \frac{\frac{db^3}{12}}{b/2} = \frac{db^3}{\cancel{12}_{6}} \times \frac{2}{b}$$

$$= \frac{db^2}{6}$$

Direct stress on the column due to load $\sigma_0 = \frac{P}{A} = \frac{P}{bd}$

Bending stress at any point of the column section at a distance 'y' from y-y axis.

$$\sigma_b = \frac{M}{Z} = \frac{Pe}{\frac{db^2}{6}} = \frac{6Pe}{db^2}$$

$$= \frac{6Pe}{bd \cdot b} = \frac{6Pe}{Ab}$$

Total stress $= \sigma_0 \pm \sigma_b$

$$= \frac{P}{A} \pm \frac{6Pe}{Ab}$$

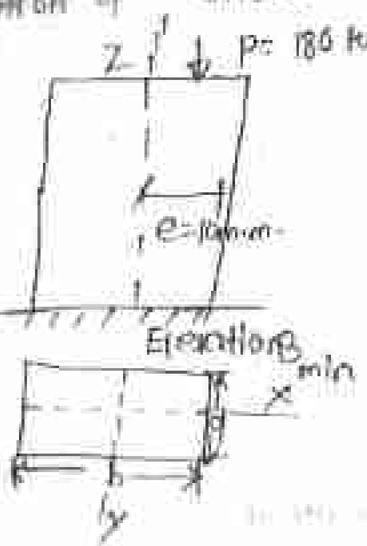
$$= \frac{P}{A}\left(1 \pm \frac{6e}{b}\right)$$

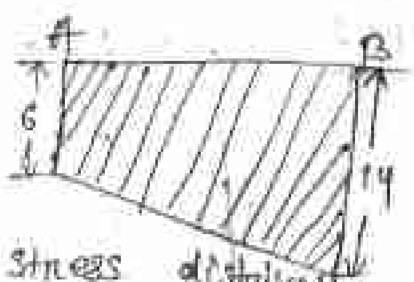
$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right)$$

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

18 A rectangular strut is 150 mm and 120 mm thick. It carries a load of 180 kN at an eccentricity of 10 mm in a plane bisecting the thickness. Find the max^m and min^m intensities of stress in the section.

Step-I




Stress distribution diagram.

Step-II

Load (P) = 180 kN = 180 × 10³ N.

eccentricity (e) = 10 m·m.

Width (b) = 150 m.m.

depth (d) = 120 m.m.

Area of the Section (A) = b × d = 150×120

= 18000 $mm^2$

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right)$$

$$= \frac{180 \times 10^3}{18000}\left(1 + \frac{6 \times 10}{150}\right)$$

$$= 14 \, N/mm^2 = 14 \, mpa$$

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

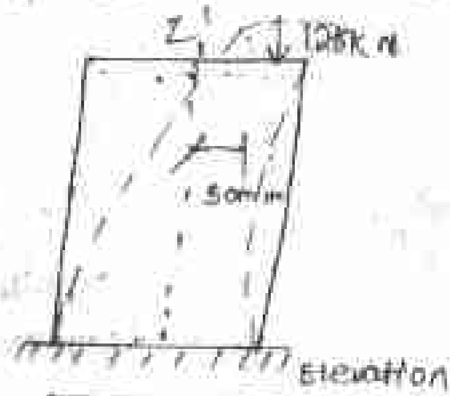
$$= \frac{180 \times 10^3}{18000}\left(1 - \frac{6 \times 10}{150}\right) = 6 \, N/mm^2$$

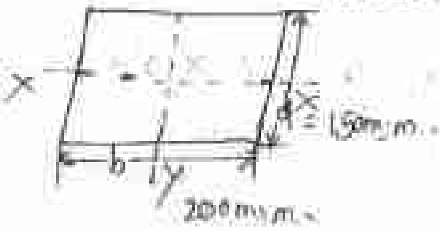
$$= 6 \, mpa$$

5 Jan 2021

A rectangular column 200 m.m which are 150 m thick is carrying a vertical load of 120 KN at an eccentrical of 150 m.m. bending the thickness determine the max$^{m}$ and min$^{m}$ intensities of stress in the section.

Step - 1



Step - II  Area of the section = $b \times d$

$= 200 \times 150$

$= 30000 \ mm^2$

Maximum stress = $\sigma_c + \sigma_b$

$= \frac{P}{A} + \frac{M}{Z}$

$= \frac{P}{A}\left(1 + \frac{6e}{b}\right)$

$= \frac{120 \times 10^3}{30000}\left(1 + \frac{6 \times 50}{200}\right)$

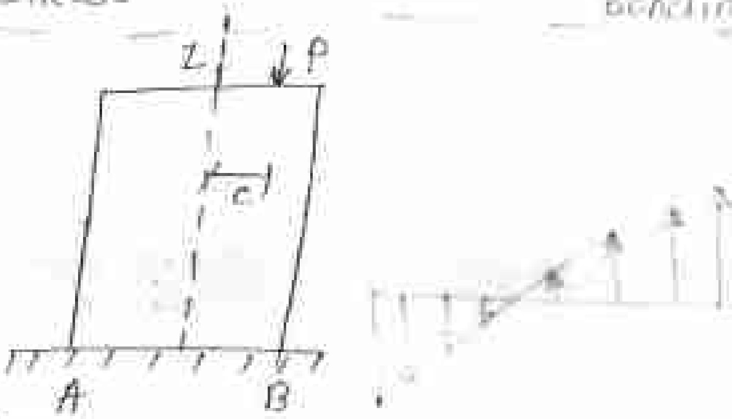
$= 16 \ mpa$

$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$

$= \frac{120 \times 10^3}{30000}\left(1 - \frac{6 \times 50}{200}\right)$

$= -2 \ mpa$

Stress distribution diagrams for Direct and bending stress

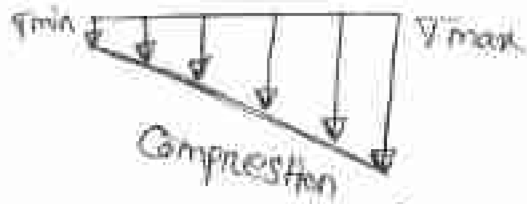



$\sigma_{min} = \sigma_o - \sigma_b$  $\sigma_{max} = \sigma_o + \sigma_b$

Case - 1  $\sigma_o > \sigma_b$

$\sigma_{max} = (+ve)$

$\sigma_{min} = (-ve)$




Elevation

Plan


stress - distribution dia.

## Case - II

$\sigma_a = \sigma_b$

$\sigma_{max} = \sigma_a - \sigma_b$

$= 2\sigma_a$ or $2\sigma_b$

$\sigma_{min} = \sigma_a - \sigma_b$

$= 0$

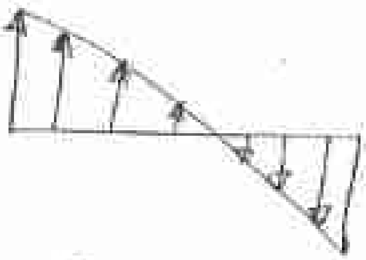

## Case - III

$\sigma_a < \sigma_b$

$\sigma_{max} = \sigma_a + \sigma_b$

$\sigma_{max} = +ve$

$\sigma_{min} = \sigma_a - \sigma_b = -ve$

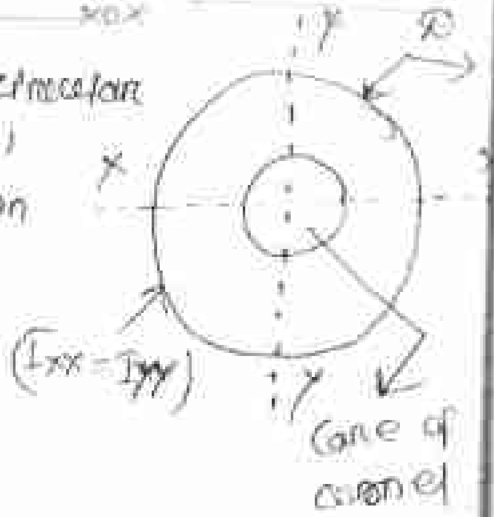


Partly tension and partly compression.

6 Jan 2021

## Limit of eccentricity of circular section

Let us consider a circular section of diameter 'D'




We know that the section modulus.

$$Z = \frac{I_{xx} \text{ or } I_{yy}}{y}$$

$$\therefore y = \frac{D}{2}$$

$$I_{xx} = \frac{\pi}{64} D^4$$

$$Z = \frac{I_{xx}}{y} = \frac{\frac{\pi}{64} \times D^4}{\frac{D}{2}} = \frac{\pi}{32} D^3$$

Area of circular section

$$A = \frac{\pi}{4} D^4$$

For no tension condition :-

$$e \leq \frac{Z}{A}$$

$$e \leq \frac{\frac{\pi}{32} D^3}{\frac{\pi}{4} D^2}$$

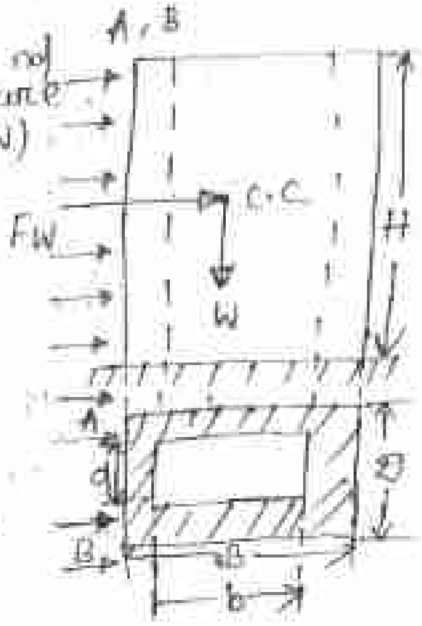
$$\leq \frac{\pi}{32} D^3 \times \frac{4}{\pi D^2}$$

$$\leq \frac{D}{8}$$

New chapter

## CHIMNEY

Stress due to wind pressure:




Direct stress due to self wt $= (\sigma_d) = \frac{P}{A}$

$P \rightarrow$ compressive load due to self-wt.

$[P = wAL]$ kN

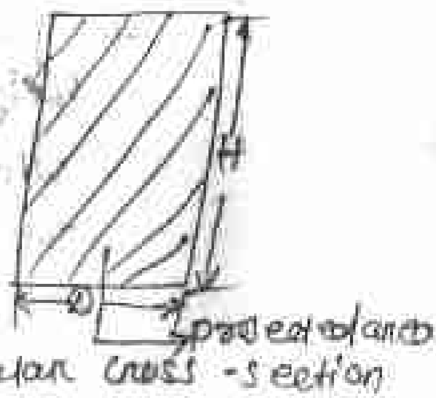
where $w \rightarrow$ weight density of chimney material (kN/m²)

$A \rightarrow$ Crosssectional area of chimney

$H = L \rightarrow$ Length of the chimney

$\sigma_d = \frac{P}{A} = \frac{w\cancel{A}H}{\cancel{A}} = wH$

**Wind force:-**




$F_W = P_W \times$ Projected area $\times$ coefficient of wind resistance (C)

$C \rightarrow \frac{2}{3} = 0.6$ for circular cross-section

$C = 1$ for flat surface.

$F_W = C \times P_W \times$ projected area

$= 1 \times P_W \times (D \times H)$

$= P_W \times D \times H$ kN.

Bend moment @ base

$$M = \left(F_\omega \times \frac{h}{2}\right) kN\text{-}m$$

$$\sigma_b = \frac{M}{Z_{yy}}$$

Section modulus @ y-axis

$$\Rightarrow Z_{yy} = \frac{I_{yy}}{y} \left[y = \frac{B}{2}\right]$$

$$\sigma_b = \frac{M}{Z_{yy}} \ kN/m^2$$

$$\sigma_{max} = \sigma_d + \sigma_b$$

$$\sigma_{min} = \sigma_d - \sigma_b$$

Q. A masonary wall 10m high 3m wide and 1.5 m thick is subjected to wind pressure 1200 N/m² find the max^m and min^m intensities of stress at base if the unit wt of the masonary is 20 kN/m³

Step-I

Data given :-

height of the wall (h) = 10m.

wide of wall (b) = 3m.

Thickness of wall (t) = d = 1.5m.

unit wt of the masonary (w)

$= 20 kN/m^3$

$= 20 \times 10^3 \ N/m^3$

wind pressure $(P_w)$ = $1200 \ N/m^2$

Step-II :- Direct stress $(\sigma_d) = \frac{P}{A}$

$$= \frac{wAh}{A} = wH$$

$= 20 \times 10^3 \times 10$

$= 200 \times 10^3 \ N/m^2$

Step-III bending stress $\sigma_b = \frac{M}{Z_{yy}}$

Wind Force (Fw) :-

$F_w = C \times p_w \times$ projected Area

$= 1 \times 1200 \times D \times H$

$= 1 \times 1200 \times 1.5 \times 10$

$= 18 \times 10^3 N$

Moment @ base, $M = F_w \times H/2$

$= 18 \times 10^3 \times \frac{10}{2}$

$= 90 \times 10^3 \ N/m$

section modulus $Z_{yy} = \frac{I_{yy}}{y} = \frac{\frac{db^3}{12}}{\frac{b}{2}}$

$= \frac{db^2}{6}$

$= \frac{1.5 \times 3^2}{6} = 2.25 m^3$

$\sigma_b = \frac{90 \times 10^3}{2.25} = 40 \times 10^3 \ N/m^2$

$\sigma_{max} = \sigma_d + \sigma_b$

$= 200 \times 10^3 + 40 \times 10^3$

$= 240 \times 10^3 \ N/m^2$

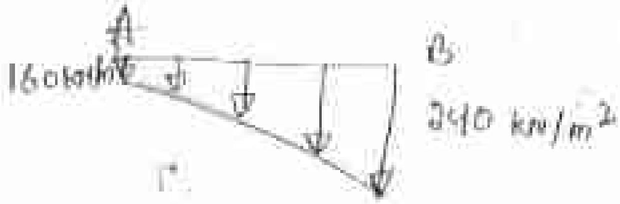
$= 240 \ kN/m^2$

$\sigma_{min} = \sigma_d - \sigma_b$

$= 200 \times 10^3 - 40 \times 10^3$

$= 160 \times 10^3 \ N/m^2$

$= 160 \ kN/m^2$



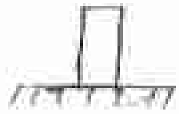

8 June 2021

DAMS

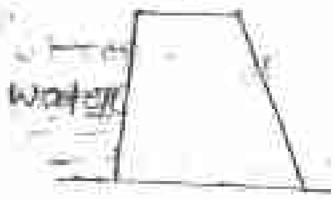
> A dam is constructed to store large quantity of water which is used for purposes of irrigati-on and power generation.

> A dam may be any cross-section. The following types dams are used in now a days.

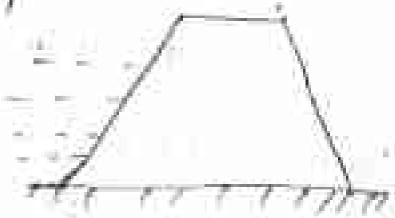
(1) Rectangular Dam



(2) Trapizoidal dams having water face vertical



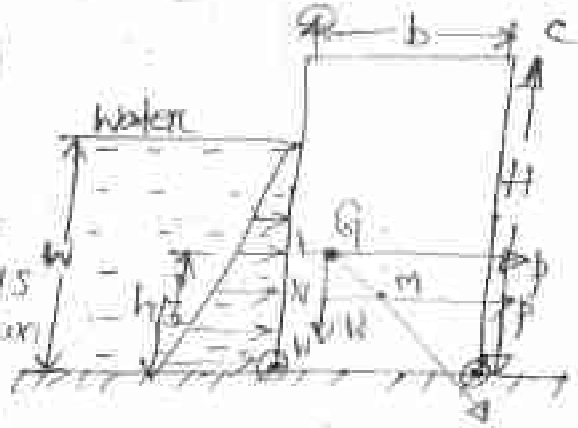

(3) Trapizoidal dams having water face inclined



(4) Rectangular Dams :-

## Rectangular Dams:-

Consider a unit length of a rectangular dam retaining water on one face of its vertical side as shown in fig.




Let $b \rightarrow$ width of the dam  A = Toe

$H \Rightarrow$ height of the dam.  B = heel

$\rho \rightarrow$ specific wt of the dam masonry

$h \rightarrow$ Height of water retained by dam.

$w \rightarrow$ specific wt of water.

Weight of the dam per unit length

$$W = \rho b h$$

The weight 'W' will act through centre of gravity of dam section.

The intensity of water pressure will be zero at the water surface and will increase by a straight line Low to wh at the bottom.

Thus avg. intensity of pressure on the fall of the dam

$$P_{avg} = \frac{0 + wh}{2} = \frac{wh}{2}$$

Total pressure per unit length of the dam $= \frac{1}{2} \times wh \times h = \frac{wb^2}{2}$

This water pressure acts at a height of h/3 from The bottom of the dam.

Now The resultant of water pressure and weight of the dam will be given by $R = \sqrt{P^2 + W^2}$

Let $x$ be the horizontal distance betn the centre of gravity of the dam and the point through which the resultant R cuts the base

From similar triangles.

$$\frac{JK}{LJ} = \frac{NM}{LN}$$

$$\Rightarrow \frac{x}{h/3} = \frac{P}{W}$$

$$\Rightarrow x = \frac{P}{W} \times \frac{h}{3}$$

Let d be the distance betn the toe of dam 'A' where the resultant cut the base.

$$d = AJ + JK$$

$$= \frac{b}{2} + x$$

$$= \frac{b}{2} + \left(\frac{P}{W} \times \frac{h}{3}\right)$$

eccentricity of resultant.

$$e = d - b/2$$

Mangitude of moment

$$M = W \cdot e$$

$$I = \frac{1 \times b^3}{12} = \frac{b^3}{12}$$

$$y = \frac{b}{2}$$

$$Z = \frac{I}{y} = \frac{\frac{b^3}{12}}{b/2} = \frac{b^3}{12} \times \frac{2}{b} = \frac{b^2}{6}$$

$$\therefore \sigma_b = \frac{M}{Z} = \frac{We}{\frac{b^2}{6}} = \frac{6We}{b^2}$$

$$\text{Direct stress} = \frac{\text{Weight of dam}}{b \times 1}$$

$$= \frac{W}{b}$$

$$\sigma_{max} = \frac{W}{b} + \frac{6We}{b^2}$$

$$= \frac{W}{b}\left(1 + \frac{6e}{b}\right)$$

$$\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$$

9 June 2021

**10** A Concrete dam of rectangular section 15 m. high and 6m. wide contains water upto height of 13m. find

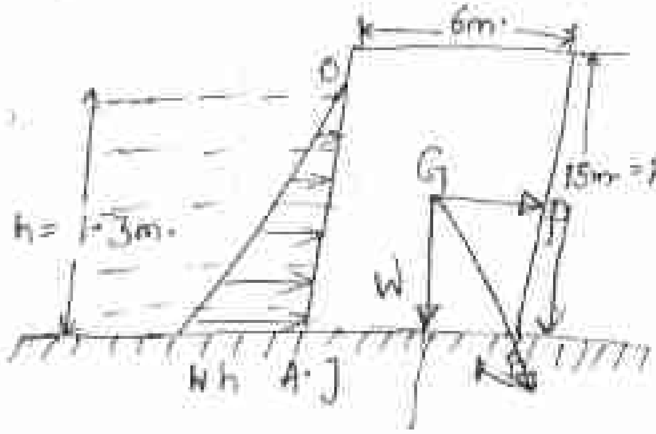
(i) Total pressure per m Length of the dam

(ii) The point where the resultant cut the base

(iii) $Max^m$ and $min^m$ intensities of stresses at the base

Assume unit wt of water and Concrete as 10 and 25 KN/m3

Step - I




Step - II

Width of the dam (b) = 6m.

Height the dam (H) = 15m.

unit wt. of water (w) = 10 kN/m³

unit wt. of Concrete $\rho$ = 25 kN/m³

(I) Total pressure per mt length of dam :-

$$P = \frac{1}{2} \times wh \times h$$

$$= \frac{1}{2} wh^2$$

$$= \frac{1}{2} \times 10 \times 13^2 = 845 \text{ kN}$$

(II) point where the Resultant cuts the base :-

Weight of the dam per mt length

$$W = \rho \times b \times H = 25 \times 6 \times 15 = 2250 \text{ kN}$$

$$x = \frac{P}{W} \times \frac{h}{3}$$

$$= \frac{845}{2250} \times \frac{13}{3} = 1.627 \text{ m.}$$

(III) Max^m and min^m intensities of stress at base.

eccentricity of the resultant

$e = x = 1.627m.$

$$\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$$

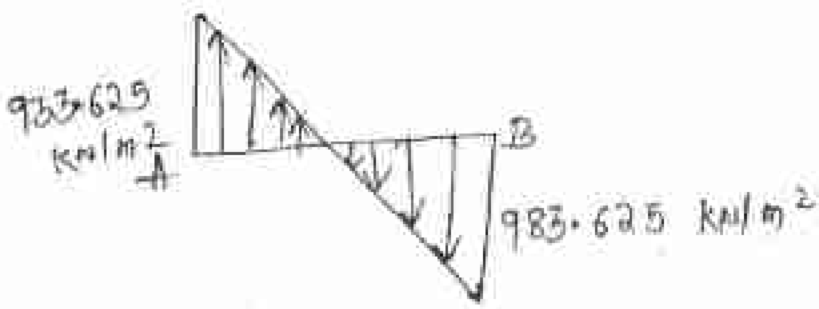
$$= \frac{2250}{6}\left[1 + \frac{6 \times 1.623}{6}\right]$$

$$= 983.625 \text{ KN/m}^2$$

$$\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$$

$$= \frac{2250}{6}\left[1 - \frac{6 \times 1.623}{6}\right]$$

$$= -233.625 \text{ KN/m}^2$$




## Trapezoidal dams with water face vertical :-

$\rho = \rho g h$

$\boxed{P = wh}$

$h = 0$

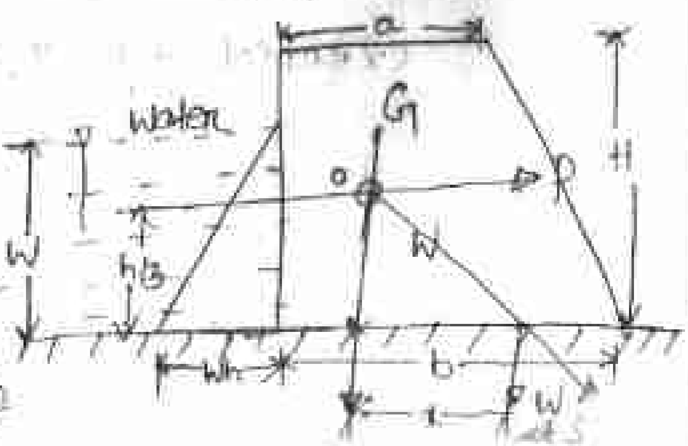
$P = 0$

$h = h$

$P = wh$

$F$ = area under pressure distribution diagram

$= \frac{1}{2} wh \times h = \frac{wh}{2}$



Consider a unit length of trapezoidal dam having its water face vertical as shown in fig.

Let $a$ = Top width of the dam

$b$ = bottom width of the dam

$H$ = Height of the dam

$\rho$ = unit wt of dam masonary

The weight of the dam per unit length

$$= \rho \times \left(\frac{a+b}{2}\right) \times H$$

Total pressure on a unit length of dam

$$P = \frac{1}{2} \times wh \times h$$

$$= \frac{wh^2}{2}$$

11 January 2021

Wt of the dam per unit length

$$= \rho \times \frac{1}{2}(a+b) \times H$$

Total pressure force exerted by water

$$\rho_M = \frac{W}{V}$$

$$\Rightarrow W = \rho_m V$$

$$= \rho_m \times A \times 1$$

$$= \rho_M \times A$$

$$= \rho_M \times \frac{1}{2}(a+b) \times H$$

kN/m

The horizontal distance between c.g of the dam and the point at which the resultant cuts the base ($x$)

$$\Sigma M_J = 0$$

$$P \times h/3 - W \times x = 0$$

$$\Rightarrow P \times \frac{h}{3} = W x$$

$$\Rightarrow x = \frac{P}{W} \times h/3$$

The distance bet 'A' and the resultant cut the base 'd'

$$d = AK + KJ$$

$$= AK + \left(\frac{P}{W} \times \frac{h}{3}\right)$$

$$AK = \frac{A x_1 + A x_2}{A_1 + A_2}$$

$$\bar{x} = \frac{a^2 + ab + b^2}{3(a+b)}$$

$$\text{Eccentricity } (e) = d - \frac{b}{2}$$

Max$^m$ stress at 'B'

$$\sigma_{max} \text{ at B} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$$

$$\sigma_{min} \text{ at A} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$$

19 A concrete dam of trapezoidal section having water on vertical force is 16m high. The base of the dam is 8m wide and top 3m wide. Find the

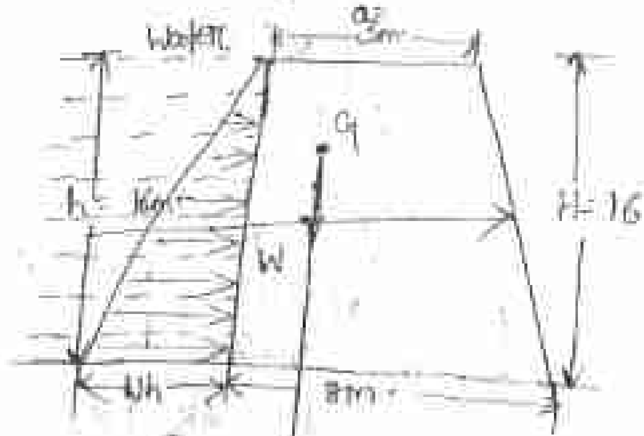
(a) resultant force thrust on the base per m length of dam.

(b) point where the resultant thrust cuts the base

(c) Intensities of $max^m$ and $min^m$.
Take unit wt of concrete as 25 kN/m³ and the water level coinciding with the top of the dam.

Soln

Step - 1




Step - II The top width of the dam (a) = 3m.
Bottom width of the " (b) = 8m.
Height of the dam (H) = 16m.
Height of the water retained by the dam (H) = 16m.
Unit wt of concrete ($\gamma$) = 25 kN/m3
Unit wt of water w = 9.81 kN/m³

Step - III The resultant thrust on the base per mt length :—
weight of the dam per unit length

$$W = \gamma \times \left(\frac{a+b}{2}\right) \times H$$

$$= 25 \times \left(\frac{3+8}{2}\right) \times 16 = 2200 \text{ kN}$$

Water force per mt length of dam

$$= \frac{1}{2} wh^2 = \frac{1}{2} \times 9.81 \times 16^2$$

$$= 1255.68 \text{ kN}$$

$$R = \sqrt{P^2 + W^2}$$

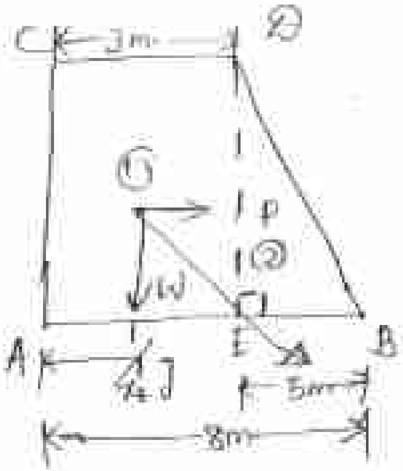
$$= \sqrt{(1255.68)^2 + (2200)^2}$$

$$= 2533.12 \text{ kN}.$$

m3
th

step- IV The point where the resultant cuts the base.

Taking moment area about A




16

$$\left[(16\times3)+\left(\frac{1}{2}\times5\times16\right)\right]\times AJ$$

$$= \left(3\times16\times\frac{3}{2}\right)+\frac{1}{2}\times5\times16\times\left(3+\frac{1}{3}\times5\right)$$

$$\Rightarrow 88\,AJ = 258.7$$

$$AJ = \frac{258.7}{88} = 2.94\text{ m.}$$

3

or $AJ = \dfrac{a^2+ab+b^2}{3(a+b)} = \dfrac{3^2+3\times5+5^2}{3(3+5)}$

se

$$= 2.04$$

The horizontal distance $bet^n$ the centre of gravity of the dam section and the poin where the resultant cut the base. $x = \frac{P}{W}\times\frac{h}{3} = \frac{1255.68}{2200}\times\frac{16}{3}$

$$x = 3.04\text{m.}$$

$$d = AJ + x = 2.94 + 3.04 = 5.98$$

Intensities of $max^m$ and $min^m$ stress at eccentricity $(e) = d - \frac{b}{2}$

$$= 5.98 - \frac{8}{2} = 1.98$$

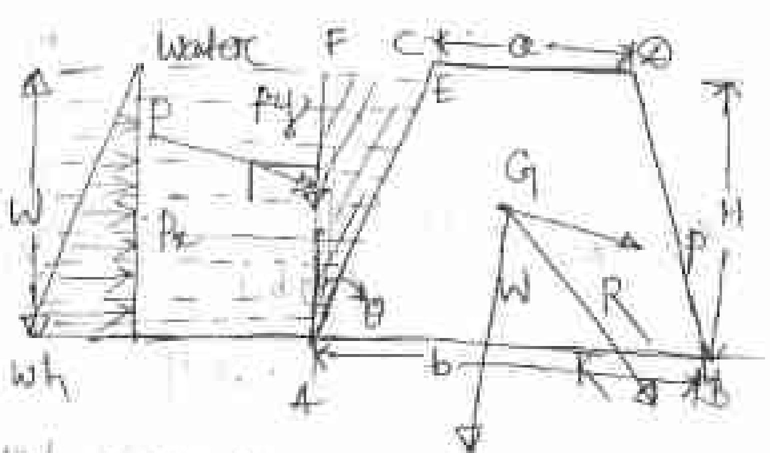
$\sigma_{max} = \frac{W}{b}\left(1+\frac{6e}{b}\right)$

$= \frac{2200}{8}\left(1+\frac{6\times1.98}{8}\right) =$

$\sigma_{min} = \frac{W}{b}\left(1-\frac{6e}{b}\right)$ kN/m²

$= \frac{2200}{8}\left(1-\frac{6\times1.98}{8}\right) = -133.4$ kN/m²

Trapezoidal dams with water fall inclined




$P_x = P\cos\theta$

$P_y = P\sin\theta$

Consider a unit length of dam trapezoidal in section as shown in the figure having water face inclined.

Let $a \rightarrow$ Top width of the dam.

$b \rightarrow$ bottom width of the dam

$H \rightarrow$ Height of the dam

$S \rightarrow$ unit wt of dam masonary

$h \rightarrow$ Height of water retained by the dam

$w \longrightarrow$ Unit width of water.

$\theta \longrightarrow$ Inclination of water face with vertical

So length of sloping side 'AE' which is subjected to water pressure

$(AE = l)$

$$\cos\theta = \frac{AF}{AE}$$

$$\Rightarrow \cos\theta = \frac{h}{l}$$

$$\Rightarrow l = \frac{h}{\cos\theta}$$

weight of the dam per unit length

$$W = \gamma \times \left(\frac{a+b}{2}\right) \times h$$

So the intensity of water pressure will be zero at the water surface and will increase at the bottom $wh$

Total pressure force on a unit length of dam $P = \frac{1}{2} wh \times l$

$$= \frac{1}{2} whl$$

The pressure force acts at a height of $h/3$ from the bottom of the dam.

Horizontal component of this water pressure

$$P_H = P\cos\theta = \frac{whl}{2} \times \frac{h}{l} = \frac{wh^2}{2}$$

vertical component of this water pressure

$$P_V = P\sin\theta = \frac{whl}{2} \times \frac{EF}{l}$$

$$= \frac{w}{2} \times EF \times h$$

= weight of the wedge AFE

The distance betn centre of gravity of dam section and the point, The resultant cut the base.

$$x = \frac{P}{W} \times \frac{h}{3}$$

Total stress at the base at B

$$\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$$

Total stress the base at A

$$= \frac{W}{b}\left(1 - \frac{6e}{b}\right)$$

Q1 A water tank contains 1.3 m deep water find the pressure exerted by the water per mt length of dam.

Sol$^n$ :- Height of the water (h) = 1.3N

$$\text{pressure}(P) = \frac{F}{A}$$

$$F = PA$$

$$P = \rho g h$$

$$= wh\, wh$$

$$w = 9.81\ kN/m^3 = 9810\ kN$$

$\frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times wh \times h$$

$$= \frac{1}{2} wh^2$$

$$\frac{wh}{2} \times h$$

$$= \frac{wh^2}{2}$$

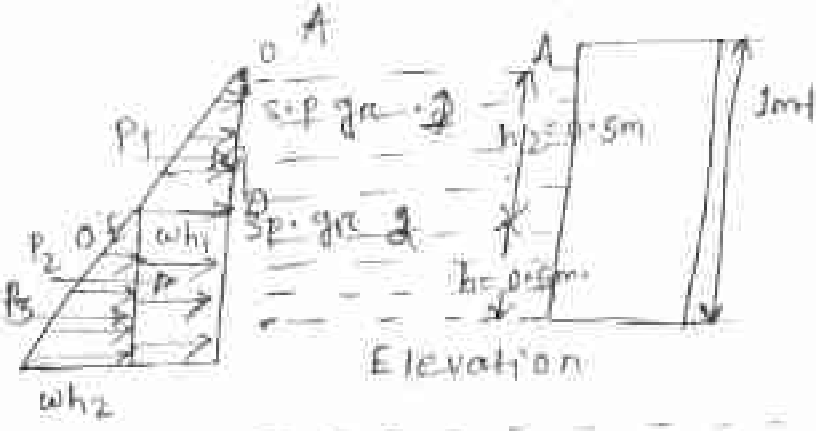
Total pressure exerted by the water

$$P = \frac{wh^2}{2} = \frac{9.81 \times 1.3^2}{2} = 8.28\ kN$$

Q2 Find the mangitude and line of action of the pressure exerted on the side of a tank. which is 1.5 m square and 1mt deep. The tank is filled half full with a

'2' while the reminder is filled with a liquid having sp. gr of '1'. Take specific wt of water $10 kN/m^3$

Sol^n :-





Data given :-

side of the square tank (a) = 1.5m.

Depth of the tank (h) = 1mt

Depth of sp gr '2' ($h_2$) = 0.5m.

Depth of s.p gr. '1' liquid ($h_1$) = 0.5m.

Unit wt of water (w) = $10 \ kN/m^3$

Magnitude of pressure :-

Intensity of pressure at 'D'

= (DE) = $w_1 h_1 = (10 \times 1) \times 0.5 = 5 kN$

$$\text{Sp gravity } 1 = \frac{\text{unit wt of liquid 1}}{\text{unit wt of water at } 4^\circ C}$$

$$\Rightarrow 1 = \frac{\text{unit wt of liquid '1'}}{10 \ kN/m^3}$$

$\Rightarrow$ unit wt of liquid. $1 = (1 \times 10) \ kN/m^3$

total pressure force of liquid (sp. gr=1)

(P) = Area of pressure diagram $\times$ length of the tank

$= \frac{1}{2} \times w_1 h_1 \times h_1 \times 1.5$

$= \frac{1}{2} w_1 h_1^2 \times 1.5$

$= \frac{1}{2} w_1 h_1^2 \times 1.5$

$= \frac{1}{2} \times 5 \times 1.5 \times 0.5$

$= 1.875$ kN.

Pressure force at 'C' due to sp. gr 1 = Area of rectangle DEFC × Length of tank

$= w_1 h_1 \times h_2 \times 0.5$

$= 5.0 \times 0.5 \times 1.5 = 3.75$ kN

Intensity of pressure at 'B' due to sp. gr '2' $= w_2 h_2 = 20 \times 0.5 = 10$ kN/m$^2$

$w_2$ = sp gr × unit wt water

$= 2 \times 10 = 20$ kN/m3

pressure force due to liquid of sp. gr '2' $P_3$ = Area of triangle EFB × Length of tank

$= \frac{1}{2} \times w_2 h_2 \times h_2 \times 1.5$

$= \frac{1}{2} \times 10 \times 0.5 \times 1.5$

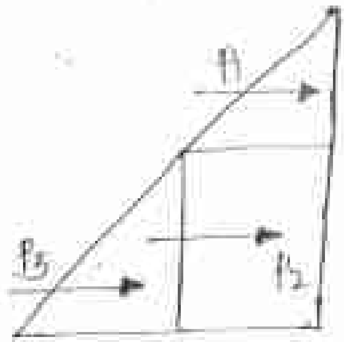
$= 3.75$ kN.

Magnitude of total force

$P = P_1 + P_2 + P_3$

$= 1.875 + 3.75 + 3.75 = 9.375$ kN

Line of action of resultant force:-




15 June 2021

Stability of dam :-

A dam should be stable under all conditions But the dam may fail.

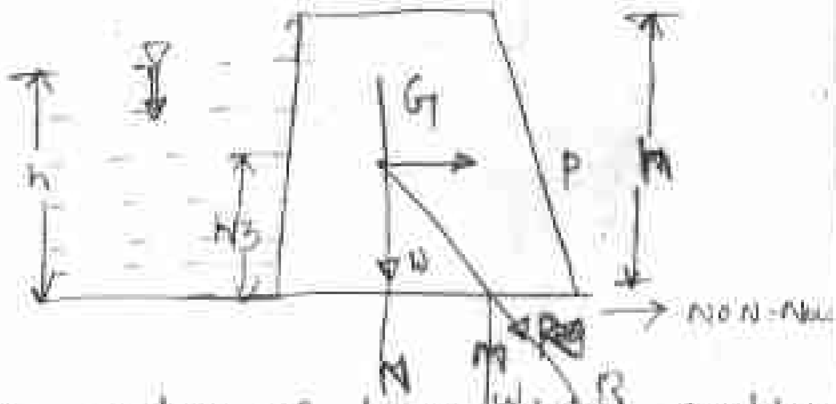
① By sliding on the soil on which it's rests.

② By over turning

③ Due to tensile stress developed.

④ Due to excessive compressive stress.

Condition to prevent the sliding of the dam :-




Consider a dam of trapezoidal section Height H and having water up to a depth of 'h' The force acting on the dam are

(i) Force due to water pressure P acting horizontally at a height of $\frac{h}{3}$ above the base.

(ii) weight of the dam 'w' acting vertically down word through the CG of dam.

So the resultant force 'P' and 'w' passing through the point 'm' the dam will be in equilibrium if a force $R^{*}$ equal to 'R' is applied at the point 'M' in the opposite direction of 'R'. Here $R^{*}$ is the reaction of the dam the reaction $R^{*}$ can be resolved in to two components. The vertical component $R^{*}$ will be equal to 'w' where as the horizontal component will be equal to frictional force at the base of the dam

$P = F_{max} = \mu w$

$$\boxed{F_{max} > \mu N}$$

Columns and Struts 05 Jan 2021

Strut :- A structural member subjected to an axial compressive force is called as strut.

=> A strut may be vertical horizontal or inclined.

=> A vertical strut is called as column which is used in building frames.

Types of column

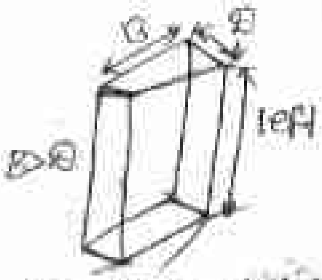
Column

Long Column $\left[\frac{l_{eff}}{D} > 12\right]$

Short Column $\left[\frac{l_{eff}}{D} < 12\right]$

Short column

① $\frac{\text{effective length}}{\text{Least lateral dimension}} < 12$




D will be taken

* Short column fails by crushing.

* Slenderness ratio < 45

$\frac{l_{eff}}{r_{min}} < 45$

$r = \sqrt{\frac{I}{A}}$

Long Column

$\frac{\text{effective length}}{\text{least lateral dimension}} > 12$

* Long column fails by buckling

* Slenderness ratio > 45

$\frac{l_{eff}}{r_{min}} > 45$

## Failure of a Column:-

When a Column is subjected to some compressive force The compressive stress induced

$$\sigma_c = \frac{P}{A}$$

P → Compressiv force

A → Cross-Sectional Area of column.

A little consideration will show that if the load is gradually increased the Column will reach a stage. when it will be subjected to the ultimate crushing stress. Beyound this stage. The column will fail by Crushing. The Load corresoponding the crushing stress is called crussing load.

Some times a compression member does not fail by crushing. but also by bending i.e buckling.. The load at which the column just to buckle is called buckling load or critical load or crippling load.

## Euler's column Theory (app. by for long column)

Assumptions of Euler's column theory:-

(1) Initially the column is perfectly straight and the load applied is linerly axial.

(2) The cross-section of the column is uniform throughout its length..

(3) The column material is perfectly elastic, homogenious and Isotropic, obeys hookes law.

(4) The length of column is very large as compared to it's cross-section.

(5) The shortening of columns due to direct compression is neglected.

(6) The failure of column occures due to buckling only.

Types of end conditions of columns

① Both ends hinged.

② Both ends fixed

③ one end is fixed and other end is hinged.

④ one end is fixed and other free.

Columns with both ends hinged:-

Consider a column AB of length L hinged at both of it's ends 'A' and 'B' carrying a critical load at 'B' let the column defects into a curved from AXB



Now consider anysection 'x' at a distance 'x' from 'A'

let $P \rightarrow$ critical load on column

$y \rightarrow$ deflection in the column at 'x'

Moment due critical load 'P'

27 June 2021

Column :- It is a structural member which is subjected to axial compressive load.

Strut :- It is a structural member which is subjected to axial compressive load. It may be horizontal or Inclined or vertical.

> The vertical strut is column.

## Slenderness ratio (S)

It is the ratio betn effective length of column and minm radious of gyration.

$$S = \frac{L_{eff}}{r_{min}}$$

$$A\, r_{min}^2 = I$$

$$r_{min} = \sqrt{\frac{I}{A}}$$

Column

Short column
$[S < 30]$

Medium Column
$[30 < S \leq 120]$

Long column
$[S > 120]$

> fails by crushing

> fails by crushing or buckling

> Long column fails buckling only.

## Euler's formula for long column :-

(P) crippling / buckling / critical

$$= \frac{\pi^2 E I_{min}}{L_{eff}^2}$$

where $E \rightarrow$ youngs modulus of column material.

$I \rightarrow (I_{xx}, I_{yy})$ min^m of $I_{xx}$ and $I_{yy}$

$I \rightarrow$ Moment of Inertia of column cross - section.

$L_{eff}$ - effective length of column.

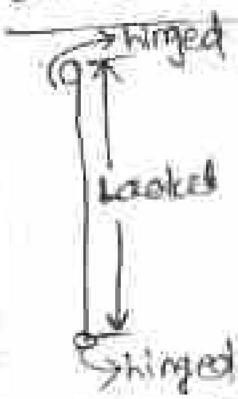
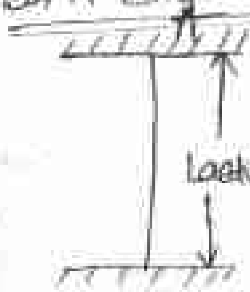
$L_{eff}$ depends on the end condition of column :-

(a) Both ends are hinged

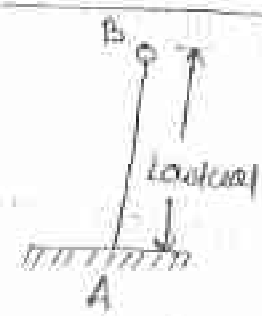
(b) Both ends are fixed

(c) One end is fixed and other end is hinged.

(d) One end is fixed or hinged and other end is free.

| Both ends are hinged | $L_{eff}$ |
|---|---|
|  | $L_{eff} = L_{actual}$ |
| Both ends fixed  | $L_{eff} = \frac{L_{actual}}{2}$ |

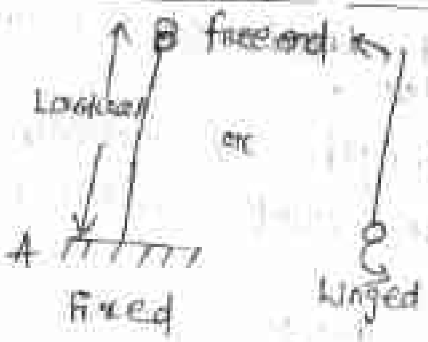
one end is fixed and other end is hinged




Left

$$L_{eff} = \frac{L_{actual}}{\sqrt{2}}$$

one end is fixed or hinged and other end is free.




Left

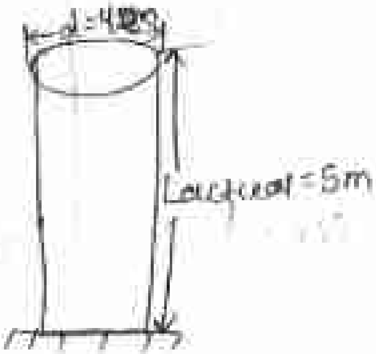
$$L_{eff} = 2L_{actual}$$

**Prob - 1** A steel rod 5m long and 4cm dia is used as a column with one end fixed and other end is free. Determine the crippling load by Euler's formula take

$$E = 2.0 \times 10^6 \ kg/cm^2$$

Soln:-

step - 1




Data given :-

Length of Column = 5m = 500 cm

Dia of column = 4 c.m.

$I_{min}$ = min$^m$ of $I_{xx}$ and $I_{yy}$

$I_{xx} = \frac{\pi}{64} \times 4^4 = 4\pi \, cm^4$

$I_{yy} = \frac{\pi}{64} \times 4^4 = 4\pi \, cm^4$

So Both the values are same, We can take any one of them.

$I_{xx} = 4\pi \, cm^4$

$E = 2.0 \times 10^6 \, kg/cm^2$

$P_{crippling} = \frac{\pi^2 EI}{L_{eff}^2}$ ——— (1)

So we know that when one end is fixed and other end is free.

$L_{eft} = 2 \, l_{actual}$

$= 2 \times 500 \, c.m = 1000 \, c.m$

$P_{crippling} = \frac{\pi^2 EI}{(1000)^2}$

$= \frac{\pi^2 \times 2.0 \times 10^6 \times 4\pi}{(1000)^2}$

$= 248.05 \, kg$ Ans

Rankine's Formula for medium column and short column:-

Rankine's Formula is given by $\frac{1}{P_R} = \frac{1}{P_C} + \frac{1}{P_E}$

$P_R \rightarrow$ crippling load or rankine's load

$P_C \rightarrow$ Crushing load $= \sigma_C A$

$P_E \rightarrow \frac{\pi^2 EI}{L_{eft}^2}$ crippling load by eluer's formula.

$$\frac{1}{P_R} = \frac{1}{P_C} + \frac{1}{P_E}$$

$$\Rightarrow \frac{1}{P_R} = \frac{P_E + P_C}{P_C P_E}$$

$$\Rightarrow P_R = \frac{P_C P_E}{P_E + P_C} = \frac{\frac{P_C \cancel{P_E}}{P_E}}{\frac{P_E + P_C}{P_E}}$$

$$= \frac{P_C}{1 + \frac{P_C}{P_E}}$$

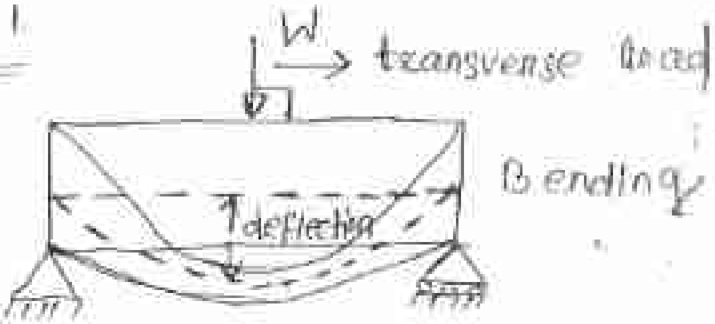
$$P = \frac{\sigma_C A}{1 + \sigma_C A \times \frac{L_{eff}^2}{\pi^2 E I}}$$

$$= \frac{\sigma_C A}{1 + \frac{\sigma_C}{\pi^2 E} \frac{A L_{eff}^2}{A K^2}}$$

NEW unit 29 Jan 2021

## slope and deflection of elastic beam

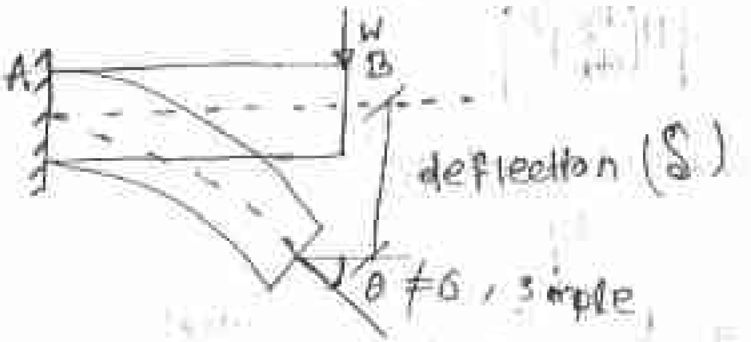
**Case - I**




**Case - II**


**Bending** :- The deviation of axis due to transverse load is called bending.

**Buckling** :- The deviation of axis due to Axial compressive load is called as buckling.




**Deflection** :- It is the linear deviation of axis under bending is called as deflection.

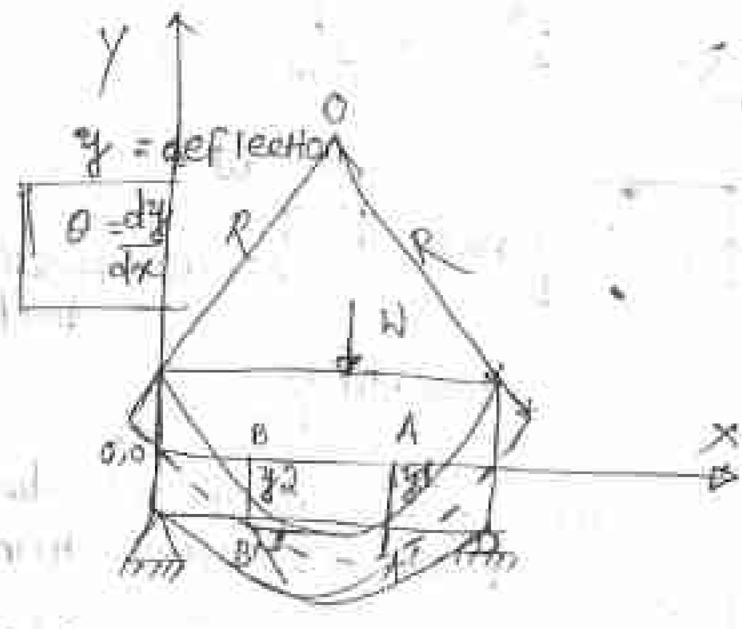
* It is denoted as $\delta$. it units is mm

Slope :- It is the angular deviation of axis under bending is called as slope.

> It is denoted by '$\theta$'.

> It's unit is radian degree.

Frame of reference :-




from elementary calculus :-

$$\frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}} \text{ ——— (i) eq}^n$$

For small deflection

$$\frac{dy}{dx} = 0$$

$$\text{so } \frac{1}{R} = \frac{d^2y}{dx^2} \text{ ——— (ii) eq}^n$$

we know that $\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$

$$\frac{M}{I} = \frac{E}{R}$$

$$\Rightarrow \frac{1}{R} = \frac{M}{EI} \text{ ——— (iii) eq}^n$$

From eqⁿ (i) and eqⁿ (iii)

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\Rightarrow \boxed{M = EI\frac{d^2y}{dx^2}}$$

$$\Rightarrow \frac{d}{dx}(M) = EI\frac{d^3y}{dx^3}$$

$$\Rightarrow \boxed{F = EI\frac{d^3y}{dx^3}}$$

$$\frac{dF}{dx} = EI\frac{d^4y}{dx^4}$$

$$\Rightarrow \boxed{w = EI\frac{d^4y}{dx^4}}$$

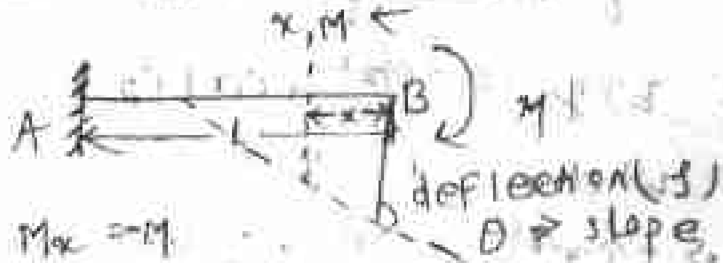
The slope and deflection of beam may be derived by following method.

(i) Double intigration method

(ii) Macaulay's method.

Double intigration method :-

Case-1




$M_x = -M$

$$M_x = EI\frac{d^2y}{dx^2}$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = -M$$

$$\Rightarrow EI\frac{dy}{dx} = \int -M\,dx$$

$$\Rightarrow EI\frac{dy}{dx} = -M\int dx$$

$$\Rightarrow EI\frac{dy}{dx} = -mx + c_1 \quad (1)$$

$$\Rightarrow EI\,y = \int -Mx\,dx + \int c_1\,dx$$

$$= -m\int x\,dx + c_1\int dx$$

$$= -m\left[\frac{x^2}{2}\right] + c_1 x + c_2$$

$$= \frac{-mx^2}{2} + c_1 x + c_2$$

for calculating the value of $c_1$ & $c_2$

$$EI\frac{dy}{dx} = -Mx + c_1$$

when $x = l$, $\frac{dy}{dx} = 0$

$$\Rightarrow 0 = -ml + c_1$$

$$\Rightarrow c_1 = Ml$$

$$EI\frac{dy}{dx} = -Mx + c_1$$

$$= -Mx + ml$$

when $x = 0$ $\frac{dy}{dx} = \max^m$

$$EI\,\theta\,\max^m = ml$$

$$\theta\,\max^m = \frac{ml}{EI}$$

$$EI\,y = \frac{-mx^2}{2} + c_1 x + c_2$$

when $x = l$, $y = 0$

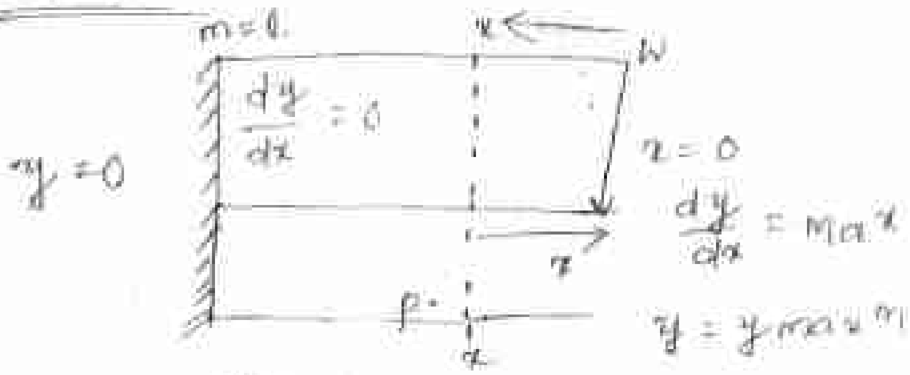
$$\Rightarrow EI\,y^{=0} = \frac{-mx^2}{2} + ml\,m + c_2$$

$$\Rightarrow 0 = \frac{-ml^2}{2} + ml \times l + c_2$$

$$\Rightarrow 0 = \frac{-ml^2}{2} + ml^2 + c_2$$

$$\Rightarrow c_2 = \frac{ml^2}{2} - ml^2 = \frac{ml^2 - 2ml^2}{2} = \frac{ml^2}{2}$$

Case-11




$$EI\frac{d^2y}{dx^2}=M_x$$

$$\Rightarrow EI\frac{d^2y}{dx}=-Wx$$

$$\Rightarrow EI\frac{dy}{dx}=\int -Wxdx$$

$$\Rightarrow EI\frac{dy}{dx}=-w\ell\int Mdx$$

$$=-w\ell\frac{x^2}{2}+C_1$$

$$\Rightarrow EI\frac{dy}{dx}=\frac{-wx^2}{2}+C_2 \longrightarrow \text{slope eq}^n$$

$$\Rightarrow EIy=\frac{-w\ell}{2}\int x^2dx-C_1\int dx$$

$$=-\frac{w}{2}\frac{x^3}{5}+C_1x+C_2$$

$$\Rightarrow EIy=\frac{-wx^3}{6}+C_1x+C_2$$

To find $C_1$ and $C_2$

$C_1$ $EI\frac{dy}{dx}=\frac{-wx^2}{2}+C_1$

when $x=l$, $\frac{dy}{dx}=0$

$$\Rightarrow 0=\frac{-wl^2}{2}+C_1$$

$$\Rightarrow C_1=\frac{wl^2}{2}$$

$$EI\frac{dy}{dx}=\frac{-wx^2}{2}+\frac{wl^2}{2}$$

$$EI\frac{dy}{dx}$$

$l^2$

When $x = 0$, $\frac{dy}{dx} = \theta_{max}$ @ B

$\Rightarrow EI\ \theta_{max}$ 'B' $= +\frac{wl^2}{2}$

$\Rightarrow \theta_{max}$ 'B' $= \frac{wl^2}{2EI}$

C2 $EIy = \frac{-wx^3}{6} = C_1x + C_2$

when $x = l$, $y = 0$

$\Rightarrow 0 = -\frac{wL^3}{6} + \frac{wL^2}{2} \times l \times C_2$

$\Rightarrow C_2 = +\frac{wL^3}{6} - \frac{wL^3}{2}$

$= \frac{wL^3 - 3wL^3}{6}$

$= \frac{-2wL^3}{3}$

$= \frac{wL^3}{3}$

$\Rightarrow EIy = \frac{-wx^3}{6} + \frac{wL^2}{2}x + \frac{wL^3}{3}$

when $x = 0$, $y = y_{max}$

$\Rightarrow EIy_{max} = \frac{-wL^3}{3}$

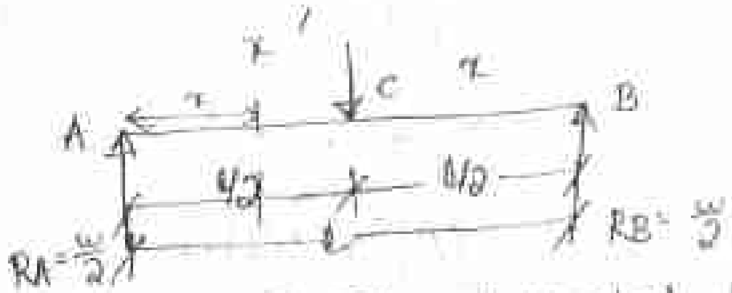
$\Rightarrow EIy_{max} = \frac{-wL^3}{3}$

$\Rightarrow \boxed{y_{max} = \frac{-wL^3}{3EI}}$

Case-III

1 Feb 2021

Slope and deflection of a simply supported beam carrying a point at its centre :-



Consider a simply supported beam 'AB' whose span is l and carrying a point load at it's centre.

Let $R_A$ and $R_B$ be the reaction at 'A' & 'B'

Taking moment at 'A'

$\Sigma M_A = 0$

OR

$T.A.M = T.C.M$

$\Rightarrow R_B \times l = W \times l/2$

$\Rightarrow R_B = \frac{W}{2}$

$T.U.L = T.D.L$

$\Rightarrow R_A + R_B = W$

$\Rightarrow R_A = W - R_B = W - \frac{W}{2} = \frac{W}{2}$

$\Rightarrow$ Consider a section x-x at a distance x from 'A'

$M_x = R_A \times x = \frac{W}{2} \times x$

$EI \frac{d^2y}{dx^2} = M_x$ (slope deflection and radious of curvature)

$EI \frac{d^2y}{dx^2} = \frac{W}{2} x$

$\Rightarrow EI = \frac{dy}{dx} = \int \frac{w}{2} x\,dx \Rightarrow EI \frac{dy}{dx} = \frac{w}{2} \int x\,dx$

$\Rightarrow EI = \frac{dy}{dx} = \frac{w}{2} \frac{x^2}{2} + C_1$

$\Rightarrow EI \frac{dy}{dx} = \frac{wx^2}{4} + C_1$ — slope eqn

$$\left[ \int x^n\,dx = \frac{x^{n+1}}{n+1} + C \right]$$

$EI \frac{dy}{dx} = \frac{wx^2}{4} + C_1$

$\Rightarrow EI\,y = \int \left(\frac{w}{4} x^2 + C_1\right) dx$

$\Rightarrow EI\,y = \frac{w}{4} \int x^2\,dx + \int C_1\,dx$

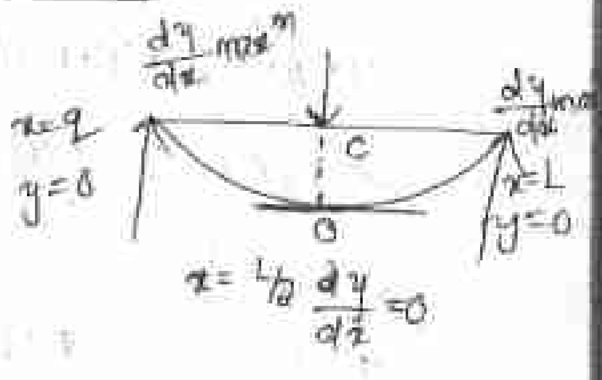
$= \frac{w}{4} \left[\frac{x^3}{3}\right] + C_1 x + C_2$

$\Rightarrow EI\,y = \frac{wx^4}{12} + C_1 x + C_2$

$\cancel{\Rightarrow EI\,y} = w_4$

$C_1$ Boundary condition :-

When $x = L/2$, $\frac{dy}{dx} = 0$




$EI \frac{dy}{dx} = \frac{wx^2}{4} + C_1$

$\Rightarrow 0 = \frac{w(L/2)^2}{4} + C_1$

$\Rightarrow 0 = \frac{wl^2}{16} + C_1$

$\Rightarrow C_1 = \frac{-wL^2}{16}$

$EI \frac{dy}{dx} = \frac{wx^2}{4} - \frac{wl^2}{16}$

when $x = 0$, $\frac{dy}{dx} = \theta_{max}$

$\Rightarrow EI\,\theta_{max} = \frac{-wL^2}{16}$

$(\theta_{max})_A = \frac{-wL^2}{16EI}$

when $x = l$ , $\frac{dy}{dx} = \theta_{max}$

$\Rightarrow EI\theta_{max} = \frac{wl^2}{4} - \frac{wl^2}{16}$

$\Rightarrow EI\theta_{max} = \frac{4wl^2 - wl^2}{16}$

$\Rightarrow EI\theta_{max} = \frac{3wl^2}{16}$

$EIy = \frac{wl^3}{12} + C_1x + C_2$

$\Rightarrow EIy = \frac{wl^3}{12} - \frac{wl^2}{16}x + C_2$

$\underline{\underline{C_2}}$

B.C

when $x = 0$ , $y = 0$

$\Rightarrow 0 = 0 + C_2$

$\Rightarrow C_2 = 0$

$EIy = \frac{wx^3}{12} - \frac{wl^2}{16}x$

$y_{max}$ L =0

when $x = l/2$ $y = y_{max}$

$EIy_{max} = \frac{w(l/2)^3}{12} - \frac{wl^2}{16}(l/2)$

$= \frac{wl^3}{8 \times 12} - \frac{wl^2}{16}(l/2)$

$= \frac{wl^3 - 3wl^3}{96}$

$= \frac{-2wl^3}{96} = \frac{-2wl^3}{48}$

$\Rightarrow EIy_{max} = \frac{-wl^3}{48}$

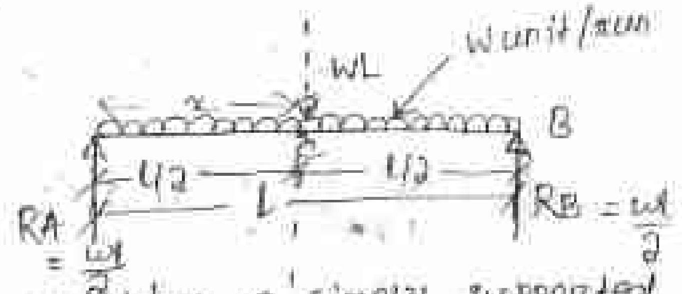
$\Rightarrow y_{max} = \frac{-wl^3}{48EI}$

- Sign indicates the deflection is downword.

Feb 2021

Case - II

Slope and deflection of a simply supported beam carrying uniformly distributed load over the entire length of beam :-




Let us consider a simply supported beam 'AB' whose length is 'l'

Let it is subjected to a load w unit/run (u.d.l) over the entire length.

Let RA & RB be the reaction at 'A' and 'B'

To find out the reaction RB

Taking moment at 'A' i.e $\Sigma M_A = 0$

or

Total or T.A.M = T.C.M

$\Rightarrow$ T.A.M = T.C.M

$\Rightarrow R_B \times l = wl \times \frac{l}{2}$

$\Rightarrow R_B = \frac{wl}{2}$

T.U.L = T.D.L

$\Rightarrow R_A + R_B = wl$

$\Rightarrow R_A = wl - R_B$

$\Rightarrow wl - \frac{wl}{2} = \frac{wl}{2}$

Let us consider a section $x-x$ at a distance $x$ from 'A'

$$M_x = R_A \times x - w \cdot x \cdot \frac{x}{2}$$

$$M_x = \frac{wl}{2} \times x - \frac{wx^2}{2}$$

From relation slope deflection and radious of curvature relationship

$$M_x = EI\frac{d^2y}{dx^2}$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = \frac{wl}{2}x - \frac{wx^2}{2}$$

$$\Rightarrow EI\frac{dy}{dx} = \int\left(\frac{wl}{2}x - \frac{wx^2}{2}\right)dx$$

$$= \frac{wl}{2}\int x\,dx - \frac{w}{2}\int x^2\,dx$$

$$= \frac{wl}{2}\left[\frac{x^2}{2}\right] - \frac{w}{2}\left[\frac{x^3}{3}\right] + C_1$$

$$= \frac{wlx^2}{4} - \frac{wx^3}{6} + C_1 \quad \text{--- (1)}$$

$$EI\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} + C_1 \quad \text{--- (1)}$$

$$\Rightarrow EIy = \int\left[\frac{wlx^2}{4} - \frac{wx^3}{6} + C_1\right]dx$$

$$= \frac{wl}{4}\int x^2\,dx - \frac{w}{6}\int x^3\,dx + \int C_1\,dx$$

$$= \frac{wl}{4}\left[\frac{x^3}{3}\right] - \frac{w}{6}\left[\frac{x^4}{4}\right] + C_1[x] + C_2$$

$$= \frac{wlx^3}{12} - \frac{wx^4}{24} + C_1x + C_2 \quad \text{--- (2) deflection}$$


$$\frac{dy}{dx} = 0$$
$$y = y_{max}$$

$\underline{\underline{C_1}}$

B.C $\Rightarrow$ when $x = l/2$ $\frac{dy}{dx} = 0$

$$EI\,\frac{dy}{dx}^{\to 0} = \frac{wl x^2}{4} - \frac{wx^3}{6} + C_1$$

$$\Rightarrow 0 = \frac{wl\left(\frac{l}{2}\right)^2}{4} - \frac{w\left(\frac{l}{2}\right)^3}{6} + C_1$$

$$\Rightarrow 0 = \frac{wl^3}{16} - \frac{wl^3}{48} + C_1$$

$$\Rightarrow C_1 = \frac{wl^3}{48} - \frac{wl^3}{16} = \frac{wl^3 - 3wl^3}{48}$$

$$\Rightarrow C_1 = \frac{-2wl^3}{48} = -\frac{wl^3}{24}$$

$$EI\,\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^3}{24}$$

$\underline{\underline{B.C}}$

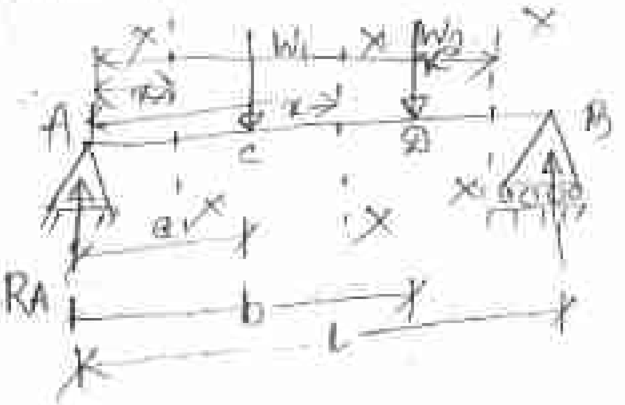
when $x=0$ $\left(\frac{dy}{dx}\right)_A = \theta_{max}$ at 'A'

$$\Rightarrow EI\ \theta_{max} \text{ at } A = -\frac{wl^3}{24}$$

$$\Rightarrow \boxed{\theta_{max} \text{ at 'A' \& 'B'} = -\frac{wl^3}{24EI}}$$

5 Feb 2021

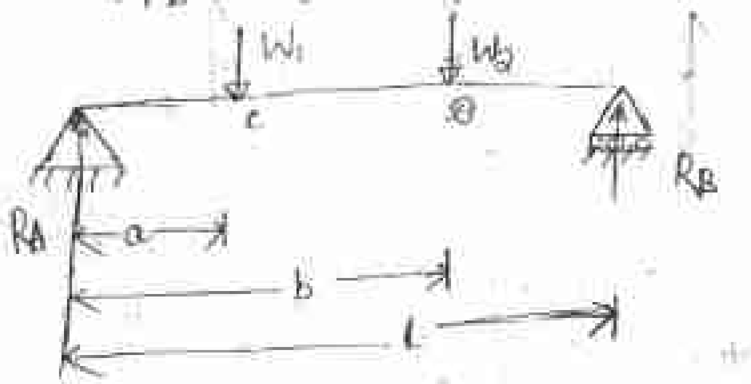
Macaulay's method to find out the slope and deflection of beam :—



$M_x = R_a \times x$

$M_x = R_a \times x - W_1$

$M_x = R_a \times x - W_1 (x-a) - W_2 (x-b)$



$M_x = \underbrace{R_a \times x}_{\text{I term}} - \underbrace{W_1 (x-a)}_{\text{II term}} - \underbrace{W_2 (x-b)}_{\text{III rd term}}$

(i) If the deflection and slope betn 'A' and 'c' is to be calculated the 1st term will be consider in the moment eq.

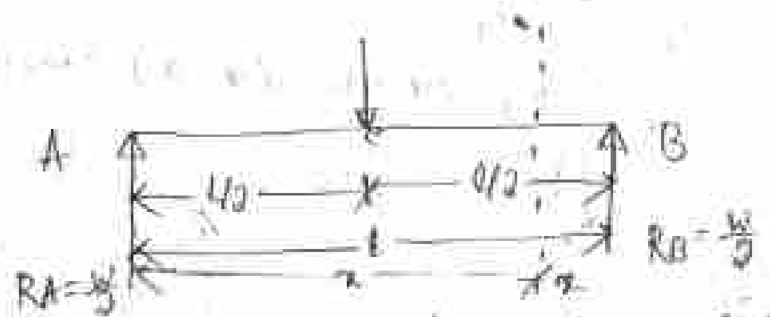
(ii) If the slope and deflection betn 'c' and 'D' is to be calculated Then up to IInd term is to be consider

(III) If the slope and deflection bet$^n$ 'O' and 'B' is to be calculated. Then upto III rd term will be considered in the moment eq$^n$.

(IV) Intigration constants should be added in the 1st term only.

$$M_x = R_a x + C_1 \,\vdots\, - W(x-a) \,\vdots\, - W_2(x-b)$$

* Slope and deflection of a simply supported beam carrying a point load at its centre.
(Macaulay's method)



Let us consider a beam simply supported beam carrying a point load W at it's centre and the length of the beam AB is 'l'.

Let it is subjected to a point load 'W' at it's centre

Let $R_A$ & $R_B$ be reaction at 'A' & 'B' respectively.

Taking moment at 'A' or $\Sigma M_A = 0$ (+)

T.A.M = T.C.M

$\Rightarrow R_B \times l = W \times \frac{l}{2}$

$\Rightarrow R_B = \frac{W}{2}$

T.U.L = T.D.L

$\Rightarrow R_A + R_B = W$

$\Rightarrow R_A = W - R_B = W - \frac{W}{2} = \frac{W}{2}$

According to macaulay's method

$M_x = R_a \times x \,|\, -W\,(x - L/2)$

$EI \frac{d^2y}{dx^2} = M_x$

$\Rightarrow EI \frac{d^2y}{dx^2} = R_a \times x \,|\, -W\,(x - L/2)$

$\Rightarrow EI \frac{dy}{dx} = \int R_a\, x\, dx \,|\, -W(x - L/2)$

$\Rightarrow EI \frac{dy}{dx} = \int R_a\, x\, dx \,|\, - \int W(x - L/2)\,dx$

$= R_a \frac{x^2}{2} \,|\, - W \int (x - \frac{L}{2})\,dx$

$= R_a \frac{x^2}{2} + C_1 \,|\, - W \frac{(x - \frac{L}{2})^2}{2}$

$EI \frac{dy}{dx} = R_a \frac{x^2}{2} + C_1 \,|\, - W \frac{(x - \frac{L}{2})^2}{2}$

6 feb 2021

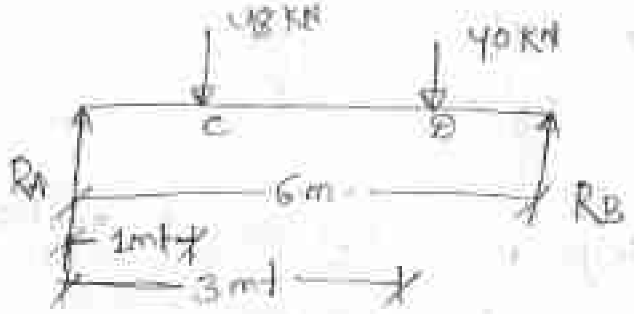
Q1 A beam of length 6m is simply supported at its ends and carries two point loads 48 kN and 40 kN at a distance of 1mt and 3mt respectively from left support find.

(i) deflection under each load

(ii) max$^n$ deflection

(iii) The point at which max$^n$ deflection occurs.

$E = 2\times10^5\ N/mm^2$ and $I = 85\times10^6$

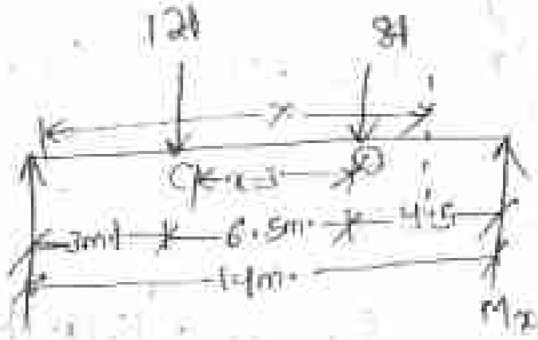


8 feb 2021

1Q A horizontal girder of steel having uniform section is 14 m. long and is simply supported at it's ends it carries point load of 12 ton and 8 ton at two point 3 m and 4.5m. from the two ends respectively

(i) $I = 160 \times 10^3\ cm^4$

(ii) $E = 2.1 \times 10^5\ kg/cm^2$




$M_x = R_A x \,|- 12(x-3)\,| - 8(x-9.5)$

**Data given**

span of the beam $(L) = 14m.$

Moment inertia $(I) = 160 \times 10^3\ cm^4$

young's modulus $(E) = 2.1 \times 10^6\ kg/m^2$

$= 2.1 \times 10^3\ ton/cm^2$

Taking moment at 'A' or $\Sigma M_A = 0$ or

$T.A.M = T.C.M$

$\Rightarrow R_B \times 14 = 12 \times 3 + (8 \times 4.5)$

$\Rightarrow R_B = 8t$

$T.U.L = T.D.L$

$\Rightarrow R_A + R_B = 12 + 8$

$\Rightarrow R_A = 20 - 8$

$\Rightarrow R_A = 12t$

As a result of this torque 'T' the shaft end 'BB' will rotate clock wise and every cross section of the shaft.

Let R = Radius of the shaft

L = Length of the shaft

$\tau$ = shear stress include at the surface of the shaft.

C = Modulus of rigidity of the material

$\phi$ = $m\angle DCD'$ equal to shear strain

$\theta$ = $m\angle DOD'$ equal to angles of twist

Now distortion the outer surface due to Torque T = DD'

shear strain at the outer surface

as = Distination per unit length

$$= \frac{DD'}{CD} = \frac{DD'}{L} = \tan\phi$$

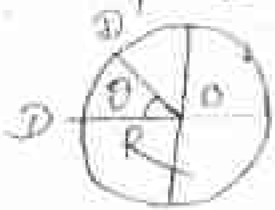
$\phi$ is very very small

so $\tan\phi = \phi$

$$= \frac{DD'}{CD} = \phi \text{ equation } (1)$$

shear strain at the outer surface $\phi$

$= \frac{DD'}{L}$



$l = r\theta$

Arc length $= \textcircled{D} \times \theta$

$DD' = R \times \theta$

put the value eq$^n$ $DD'$ in eq$^n$ ①

$\phi = \frac{R\theta}{L}$

Now the modulus of rigidity 'C' of the material of shaft.

$C = \frac{\text{shear stress produced}}{\text{shear strain produced}}$

$= \frac{\tau}{\frac{R\theta}{L}} = \frac{\tau \times L}{R\theta}$

$C = \frac{\tau L}{R\theta}$ $\boxed{\frac{C\theta}{L} = \frac{\tau}{R}}$ - eq ②

$\frac{C\theta}{l} = \frac{\tau}{R}$

$\Rightarrow \tau = \frac{RC\theta}{l}$

$\tau \propto R$

$\Rightarrow \frac{\tau}{R} = \text{constant}$

If q is the shear stress induced at a radius of 'r' from the centre.

$\frac{\tau}{R} = \frac{q}{r}$

$\frac{\tau}{R} = \frac{C\theta}{L}$

2 April 2021

(1) Q. Define poisson's ratio

Ans > It is the ratio of lateral strain to the linear strain.

> Linear strain is the primary strain which is tensile in nature then the secondary strain is compressive in nature then the secondary strain is compressive in nature.

$$\mu = \frac{\text{lateral strain or transverse strain}}{\text{Linear or primary strain}} = 1/m$$

Q.2) What is ~~the~~ point of contraflexure ?

1. In a beam the point where the bending moment changes the sign.

> At the point of contraflexure bending moment is zero.

> At the point of contra flexure the beam flexes in opposite direction.

> It is also otherwise known as point of inflexion.

Q.(3) Define Factor of safety

Ans. It is otherwise known as safety factor. It is defined as the ratio of absolute strength to actual applied load.


$F_2 \sin\theta$ → internal

No. of forces.

$$\tan\alpha = \frac{F_2 \sin\theta}{F_2 \cos\theta + F_1}$$


$F_2 \sin\theta$

$F_2 \cos\theta$

$D = s \times t$

$s = \frac{D}{t}$

$t = \frac{D}{s}$

x